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CALIBRATED NONPARAMETRIC BOOTSTRAP APPROXIMATIONS OF FINITE POPULATION *L*-STATISTICS

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Consider a study variable x taking real values in $\mathcal{X} = \{x_1, \dots, x_N\}$ in the population $\mathcal{U} = \{1, \dots, N\}$; $\mathbb{X} = \{X_1, \dots, X_n\}$ is the set of measurements of the simple random sample units $\{1, \dots, n\}$, $n < N$, drawn without replacement from $\mathcal{U}$. The L-statistic

$$L = L_n(\mathbb{X}) = \frac{1}{n} \sum_{j=1}^{n} c_{j,n} X_{j:n}$$

is a linear combination of the order statistics $X_{1:n} \le \dots \le X_{n:n}$ of $\mathbb{X}$ with the real coefficients $c_{j,n} = J(j/(n+1))$, where $J\colon (0,1) \to \mathbb{R}$, called weights. The sample mean, Gini's mean difference statistic and trimmed means are particular cases of L.

We aim to estimate the distribution function

$$F_{\mathrm{S}}(y) = \mathrm{P}\big\{\hat{\sigma}_{\mathrm{J}}^{-1}(L - \mathrm{E}L) \le y\big\}$$

of the Studentized L-statistic, where

$$\hat{\sigma}_{\mathrm{J}}^2 = \hat{\sigma}_{\mathrm{J}}^2(\mathbb{X}) = \Big(1 - \frac{n}{N}\Big) \frac{n-1}{n} \sum_{k=1}^{n} \big(L_{(k)} - \overline{L}\big)^2, \qquad \overline{L} = \frac{1}{n} \sum_{k=1}^{n} L_{(k)},$$

is the jackknife estimator of the variance $\sigma^2 = \operatorname{Var} L$. Here $L_{(k)} = L_{n-1}(\mathbb{X} \backslash \{X_k\})$, $1 \le k \le n$, are L-statistics with the weights $c_{j,n-1} = J(j/n)$, $1 \le j \le n-1$.

Usually, empirical Edgeworth expansions (EEE) and bootstrap approximations improve the normal approximation applied to $F_{\mathrm{S}}(y)$, if the sample size is not large enough. If well-correlated auxiliary information is available for all units of $\mathcal{U}$, the calibration technique [1] corrects EEEs based on the sample $\mathbb{X}$ only [2]. We construct a new calibrated nonparametric bootstrap approximation to $F_{\mathrm{S}}(y)$, and compare it with selected calibrated (parametric) EEEs and some other approximations in a simulation study.

References

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