Bandwidth analysis of optical parametric amplifier pumped by broadband pulses

VIKTORIJA TAMULIENĖ[1,*], RYTIS BUTKUS[1], AND ALGIRDAS STABINIS[1]

[1] *Laser Research Center, Vilnius University, Saulėtekio 10, Vilnius LT-10223, Lithuania*
[*] *Corresponding author: viktorija.tamuliene@ff.vu.lt*

Compiled March 26, 2020

We present analytical solution of nonlinear coupling equations for parametric down-conversion of broadband pump at large gain. The feasibility of analytical expressions is demonstrated for the case of a collinear type II parametric interaction in BBO crystal and group-velocity matching of signal and pump waves. The correlation function of amplified signal wave was obtained and it was revealed that the bandwidth of noise-seeded optical parametric amplifier increases with decrease of pump pulse duration, especially for femtosecond pump pulses.

OCIS codes: (190.0190) Nonlinear optics; (190.4970) Parametric oscillators and amplifiers; (190.7110) Ultrafast nonlinear optics.

http://dx.doi.org/10.1364/XX.XX.XXXXXX

1. INTRODUCTION

Optical parametric amplifiers (OPA) are widely regarded as devices capable to transfer energy efficiently from narrowband pump to a broadband signal [1]. With increasing focus aimed at generation of shorter signal pulses across broad spectral range, many techniques have been devised to extend OPA bandwidth [2–6]: non-collinear phase-matching, collinear phase-matching around degeneracy, especially at zero group velocity dispersion point for a signal wave, the use of angularly dispersed beams, broadband chirped pump pulses or multi-beam pumping. In this paper the dependence of OPA bandwidth on broadband pump pulse duration is analyzed for collinear phase-matching. In the case of collinear phase-matching the gain bandwidth $\Delta\Omega_{OPA}$ of OPA within the large gain and monochromatic pump low depletion approximation usually is given by [7]

$$\Delta\Omega_{OPA} = \frac{4(\ln 2)^{1/2}}{|\nu_{12}|}\left(\frac{\Gamma}{l}\right)^{1/2}, \tag{1}$$

where Γ is a parametric gain coefficient, l is the length of nonlinear crystal, and $\nu_{12} = 1/u_1 - 1/u_2$ is the mismatch of the group velocities of signal (1) and idler (2) waves which is evaluated at central frequencies ω_{10}, ω_{20}, respectively. Eq. (1) can be used for calculation of OPA bandwidth when $\Delta\Omega_{OPA}$ considerably exceeds the bandwidth $\Delta\Omega_3$ of the pump pulse, $\Delta\Omega_{OPA} \gg \Delta\Omega_3$. In this case the pump pulse can be assumed as narrowband one. For Gaussian pump pulse with an envelope $\exp(-t^2/\tau_3^2)$ the FWHM bandwidth of pump pulse is

$$\Delta\Omega_3 = 2(2\ln 2)^{1/2}/\tau_3, \tag{2}$$

here τ_3 denotes the pump pulse duration. As a result, we find

$$\frac{\Delta\Omega_{OPA}}{\Delta\Omega_3} = \frac{\tau_3}{|\nu_{12}|}\left(\frac{2\Gamma}{l}\right)^{1/2}. \tag{3}$$

For example, in the case of type II phase-matching in the KDP crystal ($\lambda_3^e = 0.532$ μm, $\lambda_1^o = \lambda_1^e = 2\lambda_3^e$) we have $\nu_{12} = 133$ fs/mm [8]. Then for $l = 1$ cm, $\Gamma = 1$ mm^{-1} and $\tau_3 = 1$ ps $\Delta\Omega_{OPA}/\Delta\Omega_3 \approx 3.3$. For type II phase-matching in the BBO crystal ($\lambda_3^e = 1.03$ μm, $\lambda_1^e = 1.60$ μm, $\lambda_2^o = 2.89$ μm) the group-velocity mismatch is larger, $\nu_{12} = -327$ fs/mm [8]. We consider type II interaction because in this case the group-velocity matching of signal and pump waves ($u_1 \approx u_3$) takes place. This type of interaction in BBO crystal is known to provide a relatively broad bandwidth [9] and is also widely exploited in commercial femtosecond tunable optical parametric amplifiers pumped by the Ti:sapphire or solid state lasers based on Yb doped active media. At $l = 1$ cm and $\Gamma = 1$ mm^{-1} a ratio $\Delta\Omega_{OPA}/\Delta\Omega_3 \le 1$ if $\tau_3 \le |\nu_{12}|(l/[2\Gamma])^{1/2} \approx 832$ fs. So, even the pump pulses with a duration $\tau_3 = 1$ ps in the last case should be considered as broadband ones.

The paper is organized as follows. The analysis of analytical solution of nonlinear coupling equations for parametric down-conversion of broadband pump at large gain is possible for group-velocity matching of signal and pump waves which is provided in Sec. II. In Sec. III the correlation function of signal wave amplified by pump pulse in the case of incoherent seed into OPA is obtained. The duration, gain and correlation properties of the signal pulse at the output of OPA are examined for type II phase matching in BBO crystal discussed above. In Sec. IV the dependence of the bandwidth of OPA on pump pulse duration is analyzed.

2. ANALYTICAL SOLUTION AT LARGE GAIN

We study the nonlinear coupling equations for the signal (1) and idler (2) fields $A_{1,2}$:

$$\frac{\partial A_1}{\partial z} + \nu_1\frac{\partial A_1}{\partial t} = i\sigma_1 A_2^* A_3, \tag{4a}$$

$$\frac{\partial A_2}{\partial z} + \nu_2\frac{\partial A_2}{\partial t} = i\sigma_2 A_1^* A_3, \tag{4b}$$

where $\nu_{1,2} = 1/u_{1,2} - 1/u_3$. z and t are the longitudinal coordinate and time, respectively, and $\sigma_{1,2} \propto \omega_{10,20}$ are the nonlinear coupling coefficients. The pump field is given by

$$A_3(t) = a_{30} \exp(-t^2/\tau_3^2), \quad \textbf{(5)}$$

where a_{30} is the amplitude. It was assumed that pump pulse is not depleted and the group-velocity dispersion is negligible which holds true for pulses longer than few tens of femtoseconds. The analytical solution of Eqs. (4) is known for group-velocity matching of signal and pump waves [10]. We note, that evolution of input signal wave in OPA being pumped by the broadband incoherent wave at the group-velocity matching of pump and idler waves was analyzed in [11, 12]. Further we take $\nu_1 = 0$, $\nu_2 = \nu$ and suppose for simplicity that only the signal wave is present at the input of OPA. In this case the solution of Eqs. (4) for the signal and idler waves is [10]:

$$\begin{aligned} A_1(t,z) &= A_{10}(t) + \frac{2\sigma_1\sigma_2}{\nu^2}\exp(-t^2/\tau_3^2)a_{30}^2 & \textbf{(6a)}\\ &\times \int_{t-\nu z}^{t} A_{10}(t_1)\exp(-t_1^2/\tau_3^2)\\ &\times (t_1 - t + \nu z)\frac{I_1(G)}{G}dt_1,\\ A_2(t,z) &= \sigma_2\frac{a_{30}}{\nu}\int_{t-\nu z}^{t} A_{10}^*(t_1)\exp(-t_1^2/\tau_3^2) & \textbf{(6b)}\\ &\times I_0(G)dt_1, \end{aligned}$$

where

$$\begin{aligned} G &= \frac{2(\sigma_1\sigma_2)^{1/2}}{\nu}a_{30} & \textbf{(7)}\\ &\times \left((t_1 - t + \nu z)\int_{t_1}^{t}\exp(-2t_2^2/\tau_3^2)dt_2\right)^{1/2}. \end{aligned}$$

Here $A_{10}(t) = A_1(t)$ at $z = 0$. Here, seed amplitude A_{10} may be both coherent or incoherent. In Sections III and IV, we study the incoherent case. $I_{0,1}$ are the modified Bessel functions. We denote the walk-off length of idler wave $L_\nu = \tau_3/|\nu|$, the nonlinear interaction length $L_n = ([\sigma_1\sigma_2]^{1/2}a_{30})^{-1} = \Gamma^{-1}$, and introduce into consideration a new variable $x = (t - t_1)/(\nu z)$. Then Eq. (6a) takes a form:

$$\begin{aligned} A_1(\xi,z) &= A_{10}(\xi) + 2\frac{z^2}{L_n^2} & \textbf{(8)}\\ &\times \int_0^1 A_{10}(\tau_3\xi - \beta\tau_3 x)\exp\left(-\xi^2 - (\xi - \beta x)^2\right)\\ &\times (1-x)\frac{I_1(G)}{G}dx, \end{aligned}$$

where $\xi = t/\tau_3$, $\beta = z/L_\nu$, and

$$G(x,\xi) = 2\frac{z}{L_n}\left(\frac{1-x}{\beta}\int_{\xi-\beta x}^{\xi}\exp(-2t'^2)dt'\right)^{1/2} \geq 0, \quad \textbf{(9)}$$

$t' = t_2/\tau_3$. We note, that $G(x,\xi) = 0$ at $x = 0, 1$. So, there exists a maximum of $G(x,\xi)$ at some point x_0, ξ_0. The equations $\frac{\partial G}{\partial \xi} = 0$ and $\frac{\partial G}{\partial x} = 0$, respectively, give:

$$\xi_0 = \beta x_0/2,\ \beta = 2\xi_0 + \left(\frac{\pi}{2}\right)^{1/2}\exp(2\xi_0^2)\mathrm{erf}(2^{1/2}\xi_0), \quad \textbf{(10)}$$

here $\mathrm{erf}(\eta) = \frac{2}{\pi^{1/2}}\int_0^\eta \exp(-\eta^2)d\eta$. The dependence $x_0 = f(\beta)$ is shown in Fig. 1, where $\beta = z/L_\nu$ and x_0 denotes the maximum of parametric gain. At $\beta^2 \ll 1$, $x_0 \approx \frac{1-\beta^2/24}{2}$.

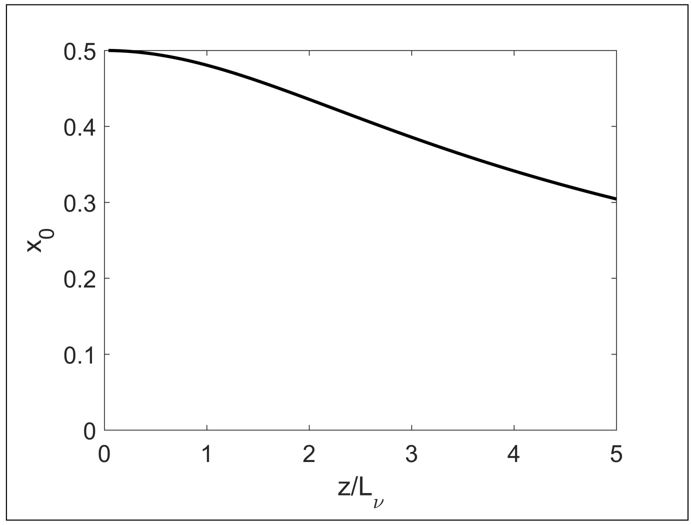

Fig. 1. Dependence $x_0 = f(z/L_\nu)$.

Further we assume a large gain $z \gg L_n$ ($G \gg 1$). In this case $I_1(G)/G \approx \frac{\exp(G)}{(2\pi)^{1/2}G^{3/2}}$. The variation of x as well as ξ at $G \gg 1$ is much faster in the numerator ($\exp(G)$) in comparison with a variation in the dominator ($G^{3/2}$). So, we take $G^{3/2}(x,\xi) \approx G^{3/2}(x_0,\xi_0)$. Then $G(x,\xi)$ can be expanded into Taylor series in the vicinity of the maximum point (x_0,ξ_0):

$$\begin{aligned} G(x,\xi) &\approx G_0(x_0,\xi_0) - p_1(x-x_0)^2 & \textbf{(11)}\\ &+ \beta p_2(x-x_0)(\xi-\xi_0) - p_2(\xi-\xi_0)^2, \end{aligned}$$

where

$$\begin{aligned} G_0 &= 2\frac{z}{L_n}(1-x_0)\exp(-\beta^2 x_0^2/4), & \textbf{(12)}\\ p_1 &= G_0\frac{1+\beta^2 x_0(1-x_0)}{2(1-x_0)^2},\ p_2 = G_0\frac{x_0}{1-x_0}. \end{aligned}$$

A substitution of Eq. (11) into Eq. (8) gives:

$$\begin{aligned} A_1(\xi,z) &\approx \kappa\exp(G_0)\int_{-\infty}^{\infty} A_{10}(\tau_3\xi - \beta\tau_3 x) & \textbf{(13)}\\ &\times \exp(-\xi^2 - (\xi-\beta x)^2 + 3\xi_0^2/2 - p_1(x-x_0)^2\\ &+ \beta p_2(x-x_0)(\xi-\xi_0) - p_2(\xi-\xi_0)^2)dx, \end{aligned}$$

where $\kappa = \frac{(z/L_n)^{1/2}}{2\pi^{1/2}(1-x_0)^{1/2}}$. We note, that $p_{1,2} \gg 1$ and therefore it was possible to change the integration limits in Eq. (13) into $-\infty, \infty$.

We define $x_1 = x - x_0$, take into account that $\xi = t/\tau_3$, $\xi_0 = t_0/\tau_3$, and rewrite Eq. (13) in the form:

$$\begin{aligned} A_1(t,z) &= \kappa\exp(G_0 - t_0^2/(2\tau_3^2) - (p_2+2)(t-t_0)^2/\tau_3^2)\\ &\times \int_{-\infty}^{\infty} A_{10}(t - 2t_0 - \beta\tau_3 x_1) & \textbf{(14)}\\ &\times \exp(-(p_1+\beta^2)x_1 + \beta(p_2+2)\\ &\times x_1(t-t_0)/\tau_3 - 2\beta x_1 t_0/\tau_3)dx_1, \end{aligned}$$

where $t_0/\tau_3 = \beta x_0/2$, see Eq. (10). This solution takes into account group velocity mismatch of signal and pump waves which appears due to interaction of signal and idler waves. By use of Eq. (6b) we can also obtain the solution for idler wave:

$$A_2(t,z) = \left(\frac{\sigma_2}{\sigma_1}\right)^{1/2} A_1^*(t,z)\exp((t^2 - t_0^2)/\tau_3^2). \quad \textbf{(15)}$$

The derived expression for signal amplitude (Eq. (14)) will be used to find the correlation function of amplified signal wave in OPA as well as the bandwidth of OPA. The Eq. (15), respectively, could be used to find the correlation function of the generated idler wave.

3. CORRELATION FUNCTION OF SIGNAL WAVE

In order to determine the bandwidth of OPA we suppose that at the input of OPA the signal wave is an incoherent broadband wave with an amplitude $A_{10}(t) = h(t)$, $\langle h(t)\rangle = 0$, where $h(t)$ is a Gaussian stochastic process and the correlation function of this process is

$$\langle h(t_1)h^*(t_2)\rangle = D\exp(-(t_2-t_1)^2/\tau_0^2), \tag{16}$$

here $\tau_0 \ll \tau_3$ is a noise correlation time. The correlation function of signal wave $F(t_1,t_2) = \langle A_1(t_1)A_1^*(t_2)\rangle$ at $\tau_0 \ll \tau_3$ by use of Eqs. (14, 16) after integration becomes:

$$\begin{aligned} F(t_1,t_2) &= \frac{\pi}{2^{1/2}}\frac{\tau_0\kappa^2 D\exp(2G_0)}{|\nu|z(p_1^2+\beta^2)^{1/2}} \\ &\times \exp\left(-\frac{\alpha_1(t_1^2+t_2^2)}{(\nu z)^2}+\frac{\alpha_2 t_1 t_2}{(\nu z)^2}-\frac{\alpha_3(t_1+t_2)}{\nu z}-\alpha_4\right), \end{aligned} \tag{17}$$

here

$$\begin{aligned} \alpha_1 &= (p_1+\beta^2)\left[1-\frac{G_0^2}{8(p_1+\beta^2)^2(1-x_0)^4}\right], \\ \alpha_2 &= \frac{G_0^2}{4(p_1+\beta^2)(1-x_0)^4}, \\ \alpha_3 &= -2x_0p_1+\frac{x_0\beta^2p_2}{2}+\frac{x_0G_0(4p_1-\beta^2p_2)}{4(1-x_0)^2(p_1+\beta^2)}, \\ \alpha_4 &= 2x_0^2p_1-\frac{x_0^2\beta^2p_2}{2}-\frac{x_0^2(4p_1-\beta^2p_2)^2}{8(p_1+\beta^2)}. \end{aligned} \tag{18}$$

As a result, for signal pulse intensity $F(t,t) = \langle |A_1(t)|^2\rangle$ we find

$$\begin{aligned} \langle |A_1(t)|^2\rangle &= v \\ &\times \exp\left(-\frac{(2\alpha_1-\alpha_2)}{\nu^2z^2}(t+t_{ps})^2+\frac{\alpha_3^2}{2\alpha_1-\alpha_2}-\alpha_4\right), \end{aligned} \tag{19}$$

here $v = \frac{\pi}{2^{1/2}}\frac{\tau_0\kappa^2D\exp(2G_0)}{|\nu|z(p_1+\beta^2)^{1/2}}$, and $t_{ps} = -\frac{\alpha_3\nu z}{2\alpha_1-\alpha_2}$ is the time delay between signal and pump pulses. Hence, group-velocity matching of signal and pump waves vanishes due to interaction of signal and idler waves within a nonlinear crystal. At $\beta^2 \ll 1$ we obtain $t_{ps}/(\nu z) \approx 1/4$. The dependence $t_{ps}/(\nu z) = f(z/L_\nu)$ is shown in Fig. 2. For $z/L_n = 5$ we have $\tau_3 = 0.2|\nu|z$, $t_{ps} \approx 0.08|\nu|z$, and we find $t_{ps}/\tau_3 \approx 0.4$.

Next we compare the signal pulse FWHM duration

$$\Delta t_s = 2|\nu|z[\ln 2/(2\alpha_1-\alpha_2)]^{1/2} \tag{20}$$

with the duration of pump pulse $\Delta t_p = (2\ln 2)^{1/2}\tau_3$. We find

$$\frac{\Delta t_s}{\Delta t_p} = \frac{2^{1/2}z/L_\nu}{(2\alpha_1-\alpha_2)^{1/2}}. \tag{21}$$

The curve $\Delta t_s/\Delta t_p = f(z/L_\nu)$ is presented in Fig. 3. At $z^2/L_\nu^2 \ll 1$ we obtain $\Delta t_s/\Delta t_p \approx (z/L_n+2)^{-1/2}$. For a fixed z value and decrease of τ_3, the ratio z/L_ν increases and so does the duration of signal pulse.

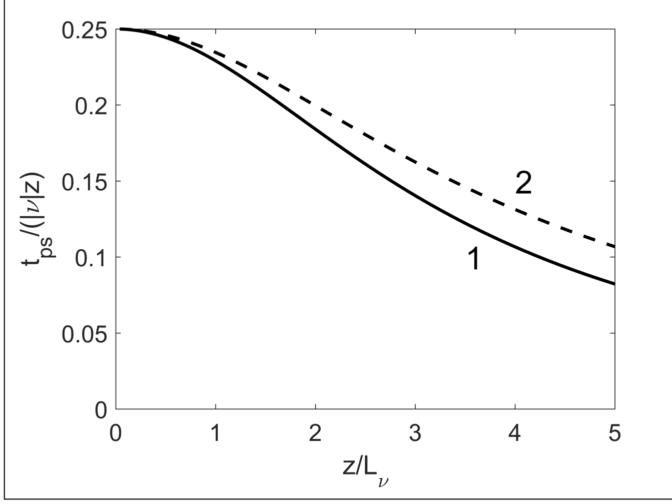

Fig. 2. The time delay between signal and pump pulses. $z/L_n = 5$ (1), $z/L_n = 10$ (2).

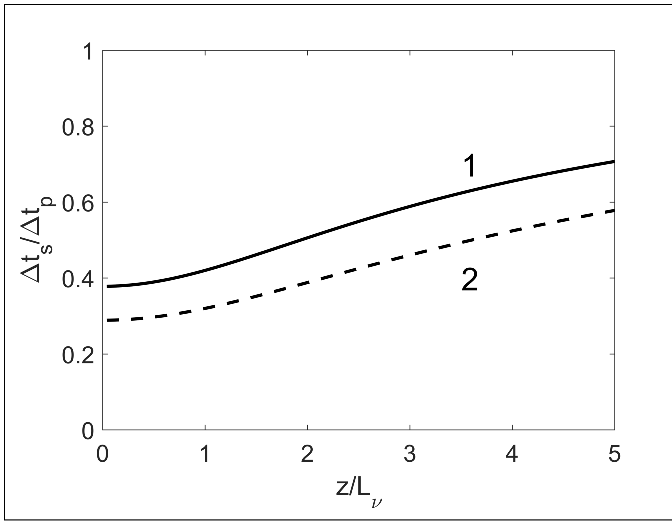

Fig. 3. Dependence of signal pulse duration on ratio z/L_ν. $z/L_n = 5$ (1), $z/L_n = 10$ (2).

At $\beta \to 0$ ($\tau_3 \to \infty$) we have $\alpha_2 \approx 2\alpha_1$, $\alpha_3 \approx \alpha_4 \approx 0$ and the intensity of signal wave at large gain in OPA pumped by monochromatic wave ($\tau_3 = \infty$) is obtained:

$$\langle |A_1(t)|^2\rangle = D\frac{\Delta\Omega_{OPA}(\tau_3=\infty)}{\Delta\Omega_0}\frac{\exp(2z/L_n)}{4}, \tag{22}$$

where $\Delta\Omega_{OPA} \ll \Delta\Omega_0$, and $\Delta\Omega_0 = 4(\ln 2)^{1/2}/\tau_0$ is the bandwidth of input signal wave. Here $\exp(2z/L_n)/4 \approx \cosh^2(z/L_n)$ at $z/L_n \gg 1$. As it is shown below (Eq. (30)), the bandwidth of OPA depends on the pump pulse duration τ_3. In this case the quantity v in Eq. (19) can be written in the form

$$v = D\frac{\Delta\Omega_{OPA}(\tau_3)}{\Delta\Omega_0}\frac{z\exp(2G_0)}{4(1-x_0)L_n(p_1+\beta^2)}. \tag{23}$$

The parametric gain at peak value of signal pulse intensity ($t = -t_{ps}$) is

$$g = \frac{z\exp(2G_0+\alpha_3^2/(2\alpha_1-\alpha_2)-\alpha_4)}{4(1-x_0)L_n(p_1+\beta^2)}, \tag{24}$$

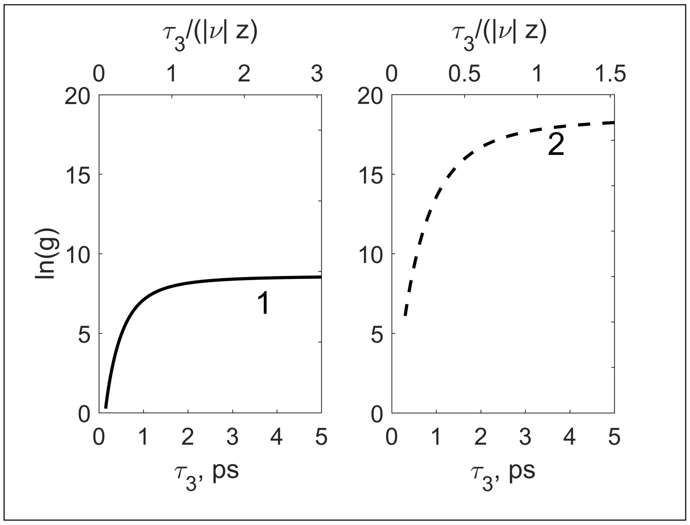

Fig. 4. Dependence of signal pulse gain on pump pulse duration τ_3 (bottom axis) and ratio $\tau_3/(|\nu|z)$ (top axis). $z/L_n = 5$ (1), $z/L_n = 10$ (2).

see Eqs. (19, 23). The dependence $\ln(g) = f(\tau_3)$ is presented in Fig. 4. We note, that with a decrease of pump pulse duration, the parametric gain decreases simultaneously due to group velocity mismatch of idler and pump pulses. To avoid the gain decrease one needs to use a shorter nonlinear crystal and increase the intensity of the pump pulse. In order to show the influence of the pulse-mismatch, we also depict the top x-axis with the normalized quantity $\tau_3/(|\nu|z)$, Fig. 4.

The degree of correlation of signal wave $\mu = F(t_1, t_2)/[F(t_1, t_1)F(t_2, t_2)]^{1/2}$ [13] is

$$\mu = \exp\left[-\frac{\alpha_2}{2}\frac{(t_1 - t_2)^2}{\nu^2 z^2}\right]. \quad \textbf{(25)}$$

We define the correlation time τ_{cor} of signal pulse at $\mu = 1/2$ and find

$$\tau_{cor} = 2(2\ln 2)^{1/2}|\nu|z/\alpha_2^{1/2}. \quad \textbf{(26)}$$

Further we compare τ_{cor} with the duration of signal pulse, see Eq. (20), and have

$$\frac{\tau_{cor}}{\Delta t_s} = [2(2\alpha_1 - \alpha_2)/\alpha_2]^{1/2}. \quad \textbf{(27)}$$

The curves $\frac{\tau_{cor}}{\Delta t_s} = f_1(z/L_\nu)$ and $\frac{\tau_{cor}}{\Delta t_s} = f_2(\tau_3)$ are presented in Figs. 5a, 5b, respectively. The results reveal an important insight into the coherence of amplified signal wave – at $\frac{\tau_{cor}}{\Delta t_s} > 1$ the signal pulse correlation time exceeds the pulse duration, and therefore a coherent signal pulse is obtained, Fig. 5a. At $z^2/L_\nu^2 \ll 1$ we find $\tau_{cor}/\Delta t_s \approx \frac{z}{L_\nu}[2(1 + 2L_n/z)]^{1/2}$. At large gain (for $z \gg L_n$), the ratio $\tau_{cor}/\Delta t_s = 1$ for $z/L_\nu \approx \frac{1}{2^{1/2}} \approx 0.7$. Thus, the coherent signal pulses are produced in OPA at $\frac{z}{L_\nu} \geq 0.7$, see Fig. 5a. With a decrease of pump pulse duration, the correlation time of signal pulse increases (Fig. 5b) but the signal gain decreases, see Fig. 4.

In order to verify the validity of analytical expressions we performed numerical simulations using the same method as in [12]. The results of numerical simulations are depicted as insets in Fig. 5a and they represent one-realization output signal spectra as well as theoretical profiles of averaged spectrum for two different ratios of z/L_ν. One can see the more random-like shape of signal spectrum for low z/L_ν ratio and a smooth envelope for high z/L_ν ratio.

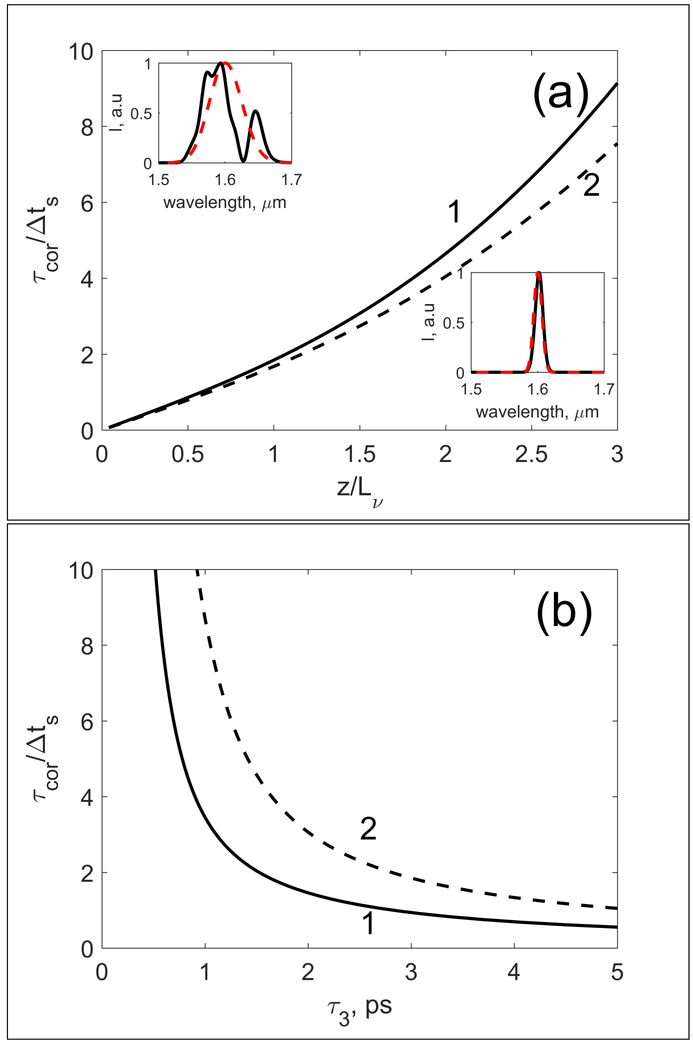

Fig. 5. (Color online) Dependences of signal pulse correlation time on ratio z/L_ν (a) and pump pulse duration (b). $z/L_n = 5$ (1), $z/L_n = 10$ (2). Insets in (a): one-realization output signal spectra $I = |S_1|^2$ obtained by numerical simulation of Eqs. (4) at $z = 0.5L_\nu$ (left inset, solid black line) and $z = 3L_\nu$ (right inset, solid black line). Here, $\tau_3 = 500$ fs, $z/L_n = 10$ and red dashed lines: theoretical profiles of averaged spectrum $\langle|S_1|^2\rangle$, Eq. (29).

We note that in a similar way it is possible to obtain the correlation function of idler wave $\langle A_2(t_1)A_2^*(t_2)\rangle$ as well as cross-correlation function of signal and idler waves $\langle A_1(t_1)A_2(t_2)\rangle$ by the use of Eqs. (6a, 6b). Due to phase conjugation of signal and idler waves in a parametric downconversion process, the following relation applies: $\langle A_1(t_1)A_2^*(t_2)\rangle = 0$. By the use of Eq. (15) we find $\langle|A_2|^2\rangle = \frac{\sigma_2}{\sigma_1}\langle|A_1|^2\rangle \exp(2(t^2 - t_0^2)/\tau_3^2)$ and the time delay t_{pi} between the idler and pump pulses is $t_{pi}/t_{ps} = (2\alpha_1 - \alpha_2)/(2\alpha_1 - \alpha_2 - 2\beta^2)$. At $(z/L_\nu)^2 \ll 1$ we obtain $t_{pi}/t_{ps} \approx 1 + (1 + z/L_n)^{-1}$, and the delay between the idler and pump pulses Δt_{pi} at large gain only slightly exceeds the delay between the signal and pump pulses Δt_{ps}.

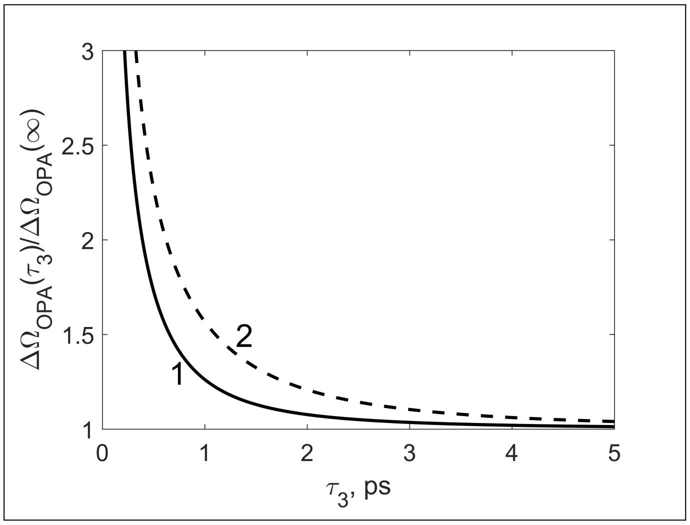

Fig. 6. Dependence of OPA bandwidth on pump pulse duration. $z/L_n = 5$ (1), $z/L_n = 10$ (2).

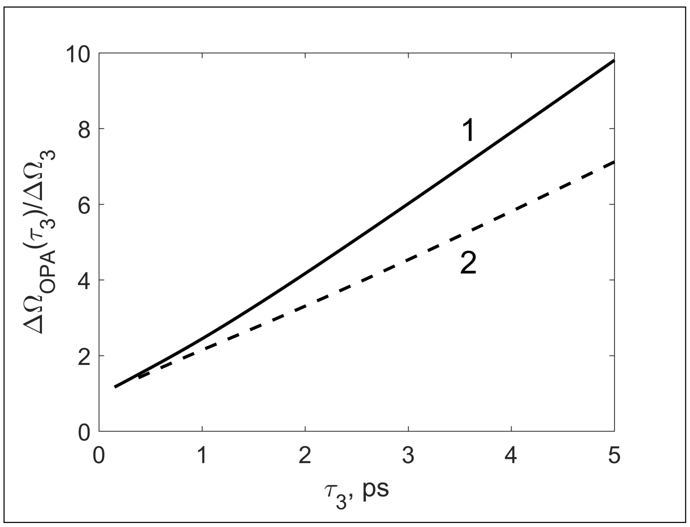

Fig. 7. Dependence of OPA bandwidth normalized to pump pulse bandwidth on pump pulse duration. $z/L_n = 5$ (1), $z/L_n = 10$ (2).

4. BANDWIDTH OF OPA PUMPED BY THE BROADBAND PULSE

In order to find the bandwidth of OPA we perform the Fourier transformation $S_1(\Omega) = \int_{-\infty}^{\infty} A_1(t)\exp(-i\Omega t)dt$ and obtain:

$$\langle |S_1(\Omega)|^2\rangle = \int\int_{-\infty}^{\infty} \langle A_1(t_1)A_1^*(t_2)\rangle \exp(-i\Omega(t_1-t_2))dt_1dt_2, \tag{28}$$

here $\Omega = \omega_1 - \omega_{10}$. As a result, by the use of Eq. (17) we have

$$\frac{\langle |S_1(\Omega)|^2\rangle}{\langle |S_1(0)|^2\rangle} = \exp[-\Omega^2\nu^2z^2(2\alpha_1+\alpha_2)] = \exp\left[-\frac{\Omega^2\nu^2z^2}{2(p_1+\beta^2)}\right], \tag{29}$$

and the bandwidth of OPA is

$$\Delta\Omega_{OPA}(\tau_3) = (8\ln(2)(p_1+\beta^2))^{1/2}/(|\nu|z). \tag{30}$$

Further we compare $\Delta\Omega_{OPA}(\tau_3)$ with a bandwidth of OPA pumped by the monochromatic wave $\Delta\Omega_{OPA}(\infty)$ (here $\beta = 0$ and $p_1 = 2z/L_n$) and find

$$\frac{\Delta\Omega_{OPA}(\tau_3)}{\Delta\Omega_{OPA}(\infty)} = \left(\frac{L_n(p_1+\beta^2)}{2z}\right)^{1/2}. \tag{31}$$

The dependence of OPA bandwidth on pump pulse duration is depicted in Fig. 6. Apparently the bandwidth of OPA increases with a decrease of pump pulse duration, especially for the femtosecond pulses. We note that simultaneously the parametric gain decreases, see Fig. 4. In order to avoid the gain decrease it is preferable to use a shorter nonlinear crystal and higher pump pulse intensity to preserve parametric gain (ratio z/L_n). The bandwidth of OPA increases with increase of parametric gain (z/L_n), see Fig. 6.

The bandwidth of OPA $\Delta\Omega_{OPA}(\tau_3)$ can then be compared to a bandwidth of pump pulse $\Delta\Omega_3$, see Eq. (2) using the following expression:

$$\frac{\Delta\Omega_{OPA}(\tau_3)}{\Delta\Omega_3} = (p_1+\beta^2)^{1/2}\frac{\tau_3}{|\nu|z} = \left(1+\frac{p_1}{\beta^2}\right)^{1/2}. \tag{32}$$

The dependence of this ratio on pump pulse duration is presented in Fig. 7. For a fixed z/L_n value the ratio $\Delta\Omega_{OPA}(\tau_3)/\Delta\Omega_3$ decreases with decrease of τ_3, and at small τ_3 values we have $\Delta\Omega_{OPA}(\tau_3) \approx \Delta\Omega_3$.

In the case of pump depletion, with a propagation in the nonlinear crystal the decrease of parametric gain as well as modifications of group delays will take place. An OPA bandwith analysis needs solution of full Eqs.(4) including equation for pump field.

5. CONCLUSIONS

The bandwidth of OPA pumped by the broadband pulses was analyzed at large gain for collinear type II phase-matching in BBO crystal ($\lambda_3^e = 1.03\ \mu$m, $\lambda_1^o = 1.60\ \mu$m, $\lambda_2^e = 2.89\ \mu$m) at group-velocity matching of signal and pump pulses in the case of incoherent seed.

The correlation function as well as degree of correlation of amplified signal pulse in OPA are obtained. It is revealed that coherent signal pulses are produced in OPA at $z/L_\nu \geq 0.7$. The group-velocity matching of signal and pump pulses vanishes during the amplification of signal pulse in OPA due to interaction with generated idler pulses in nonlinear medium.

The bandwidth of OPA increases with a decrease of pump pulse duration (especially for femtosecond pump pulses) as well as with increase of gain factor z/L_n. The decrease of pump pulse duration inflicts the decrease of the parametric gain due to gradual saturation process caused by group-velocity mismatch of idler and pump pulses. In order to avoid the gain decrease it is preferable to use a shorter nonlinear crystal and to increase the pump pulse intensity.

The obtained results might be feasible for type II OPAs pumped by few hundred femtosecond long pulses, e.g. derived from Yb:KGW or Yb:YAG lasers, as the OPA bandwidth differs significantly from the case of monochromatic pump.

6. FUNDING INFORMATION

Rytis Butkus acknowledges European Regional Development Fund (ERDF) (01.2.2-LMT-K-718).

7. DISCLOSURES

Disclosures. The authors declare no conflicts of interest.

REFERENCES

1. C. Manzoni and G. Cerullo, “Design criteria for ultrafast optical parametric amplifiers,” *J. Opt.* **18**, 103501 (2016).
2. G. M. Gale, M. Cavallari, T. J. Driscoll, and F. Hache, “Sub–20–fs tunable pulses in the visible from an 82–MHz optical parametric oscillator,” *Opt. Lett.* **20**, 1562–1564 (1995).
3. A. P. Piskarskas, A. P. Stabinis, and V. Pyragaite, “Ultrabroad bandwidth of optical parametric amplifiers,” *IEEE J. Quantum Electron.* **46**, 1031–1038 (2010).
4. R. Dabu, “Very broad gain bandwidth parametric amplification in nonlinear crystals at critical wavelength degeneracy,” *Opt. Express* **18**, 11689–11699 (2010).
5. E. Žeromskis, A. Dubietis, G. Tamošauskas, and A. Piskarskas, “Gain bandwidth broadening of the continuum-seeded optical parametric amplifier by use of two pump beams,” *Opt. Commun.* **203**, 435 – 440 (2002).
6. D. Herrmann, C. Homann, R. Tautz, M. Scharrer, P. S. Russell, F. Krausz, L. Veisz, and E. Riedle, “Approaching the full octave: Noncollinear optical parametric chirped pulse amplification with two-color pumping,” *Opt. Express* **18**, 18752–18762 (2010).
7. S. E. Harris, “Tunable optical parametric oscillators,” *Proc. IEEE* **57**, 2096 (1969).
8. A. V. Smith, computer code SNLO (AS-Photonics, Albuquerque, NM, 2012).
9. C. Schmidt, J. Buehler, A.-C. Heinrich, A. Leitenstorfer, and D. Brida, “Noncollinear parametric amplification in the near-infrared based on type-II phase matching,” *J. Opt.* **17**, 094003 (2015).
10. A. P. Sukhorukov and A. K. Shchednova, “Parametric amplification of light in the field of a modulated laser wave,” *Sov. Phys. JETP* **33**, 677 (1971).
11. A. Picozzi and P. Aschieri, “Influence of dispersion on the resonant interaction between three incoherent waves,” *Phys. Rev. E* **72**, 046606 (2005).
12. V. Pyragaite, A.Stabinis, A. Piskarskas, and V. Smilgevičius, “Parametric amplification in the field of incoherent light,” *Phys. Rev. A* **87**, 063809 (2013).
13. E. A. Saleh and M. Teich, *Fundamental of photonics* (Wiley, 2007), 2nd ed.