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FORECASTING BOND RETURNS USING ASYMMETRIC REGRESSION AND INVESTMENT MANAGEMENT

Jonas Kanapeckas
Institute of Mathematics and Informatics, Akademijos 4, 2600 Vilnius, Lithuania

Abstract

The first section of this research formulates the forecasting task important for managing investment portfolio as well as discusses certain statistical data. The second section is devoted to potential regressors frequently used to forecast risk premiums of bonds, this section extensively use the ideas presented in article [4]. The third section includes the research of asymmetry of relation between risk premiums and regressors. The fourth section is devoted to the investigation of applicability received results in practice.

INTRODUCTION

Let us assume that, at any time t, there is a set of N different bonds[1]. The price of a bond at the time t is equal to P_t^i, i=1,…,N. If, at the time t-1, we invested into bond of type i an amount of money equal to the price of the bond P_{t-1}^i, then, at the time t, our profit/loss of this investment would amount to P_t^i - P_{t-1}^i, where P_t^i means the price of an i-type bond at the time t. However, the outcome of investment is usually expressed in relative, rather than absolute, variables, such as return on investment[2]:

$$R_t^i = \frac{P_t^i - P_{t-1}^i}{P_{t-1}^i} \approx \log P_t^i - \log P_{t-1}^i \text{, } i \in \{1,\dots,N\}. \tag{1}$$

The decision to invest into bonds is followed by the consideration how the funds available should be distributed among different types of bonds. Two main factors should be taken into account: first, the utility function of the investor and expected

[1] Most often, bonds equal in credit risk are distinguished according to their term to maturity.
[2] Such approach is more convenient due to the following reasons: first, the result does not depend on the actual amount invested, second, returns on investment have better statistical properties.

returns from different types of bonds. Let $R_t^{j,i}$ denote the risk premium (or just premium) of a j-type bond with respect to an i-type bond during the period from t to t+1 ($i, j \in \{1,\ldots,N\}$, $i \neq j$):

$$R_t^{j,i} = R_t^j - R_t^i, \tag{2}$$

For the moment, let us concentrate on the forecasting of risk premiums rather than discuss the use of utility functions and premium forecasts in the formation of investment portfolios. To be more precise, the present research is devoted to empirical comparison of the possibilities to apply some time series forecasting methods for prediction of relative return on investments (risk premiums).

I. STATISTICAL DATA OF RISK PREMIUMS

For the evaluation of return on investments of various-term bonds, the data[3] of the indices of the US government securities published by Salomon Smith Barney was used. The government securities of the following terms (or their adequate indices) were used: 1 month, 1, 2, 3, 5, 10 and 30 years. They publish the values of aforementioned indices on the last day of each month. The data on 1-month-term securities is available from 1978, while on the rest - from January 1980.[4] Without going into further detail, it is possible to state that these values express the prices of financial assets on the last day of a month[5]. As the research limits itself to the data up to January 1998, following returns on investments were computed:

Term	From	To	Total
1 month	February, 1978	January, 1998	240
1,2,3,5,10 and 30 years*	February, 1980	January, 1998	216

*In two cases, according to formula (1), the received values of returns on investment for the period of two months were divided into two equal parts (see footnote 4).

[3] Their original names are as follows: Salomon Smith Barney U.S. Treasury Benchmark (On the run) Indexes[SM] and Salomon Smith Barney U.S. Treasury Bill Indexes[SM]. The data was supplied by the Bloomberg information agency.
[4] Due to unknown reason, data on the terms of 1, 2, 3, 5, 10 and 30 years for February 1980 and 1984 was not available.
[5] For details see [1].

The received values and formula (2) were used to find out monthly risk premiums for these types of bonds:

Bonds of the following terms are compared	From	To	Total
Risk premiums of 1,2,3,5,10,30-year bonds with respect to one month bond	February, 1980	January, 1998	216
Risk premiums of 2,3,5,10,30-year bonds with respect to 1-year bond	February, 1980	January, 1998	216

For the sake of simplicity, further in the text we will only refer to the risk premiums of to 1 and 30 year bonds with respect to 1 month bond.

II. ANALYSIS OF REGRESSORS

Steepness of the interest rate curve

Concept of the interest rate curve. At any time, there are abundance of various bonds with adequate interest rates on the market. In a few words, the interest rate of a bond is a price for which the lender lends, and the borrower borrows the money for a certain period. The interest rate curve is designed using terms to maturity of bonds as abscises (*x*-axis) and adequate interest rates as ordinates (*y*-axis). Therefore, the steepness of the interest rate curve is expressed as difference between the interest rates of relatively long term and short term bonds. However, in practice, bonds differ from each other not only by their terms, there are many other important characteristics which extremely complicate the estimation of the interest rate curve, its interpretation and application. One of such characteristics dividing all bonds into two different types is the payment of interest to the investor before bond maturity: on one hand, the investor periodically receives agreed interest (coupon) for the whole period of investment (coupon bonds), on the other hand, he does not receive any before maturity (bonds of such type are referred to as zero-coupon bonds). According to this characteristic, certain types of the interest rate curves are distinguished: the interest rate curve of bonds and the interest rate curve of zero-coupon bonds. Due to the fact that there are variety of bonds on the market (and at the same time, there are terms "without" bonds or bonds of the same term with different interest rates), the date for the interest rate curve are averaged and smoothed[6]. In addition to that, it is often more convenient to use the theoretical interest rate curve of zero-coupon bonds designed using specific methods and coupon bonds data. One of such methods, namely bootstrap method, was used in this research, as well.

Economical background of relation between the steepness of the interest rate curve and risk premium. The theory points out three main factors affecting the shape of the interest rate curve: they are risk premium, interest rate expectations and convexity bias[7]. In the discussions regarding the shape of the interest rate curve, terms of curve level, steepness and convexity are often used. Moreover, certain part of the interest rate curve steepness reflects the difference among the expected returns on the bonds of various terms, i.e. steep (flat) curve means large positive (small or negative) difference between the expected returns of longer term and shorter term investments.[8] As regards this relation, the present research includes its empirical analysis.

Interest rate curve in linear regression. Daily data on interest rates of bonds were taken from US Federal Reserve Statistical release.[9] Then, using one of practical methods, estimation of theoretical interest rates curve of zero-coupon bonds was carried out. The table below contains the received data:

Term	From	To	Total:
1 month	February 15, 1977	February 25, 1998	5,248
1 year	February 15, 1977	February 25, 1998	5,248
2 year	February 15, 1977	February 25, 1998	5,248
3 year	February 15, 1977	February 25, 1998	5,248
5 year	February 15, 1977	February 25, 1998	5,248
10 year	February 15, 1977	February 25, 1998	5,248
30 year	February 15, 1977	February 25, 1998	5,248

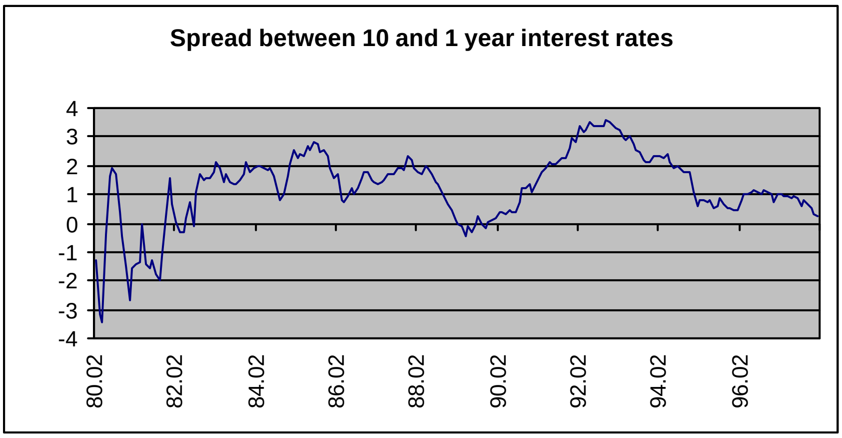
[6] For details see [2].
[7] For details see [3].
[8] It refers to the difference of monthly returns on the investment into securities of different types. E.g. Difference between the relative change in price of 1-year and 10-year bonds
[9] Federal Reserve Statistical release H.15 *Selected Interest Rates.* Internet: http://www.bog.frb.fed.us/releases/h15/. Data as well as their description are available.

Actually, this enabled us to receive the theoretical interest rate curve of zero-coupon bonds (expressed in interest rates for selected terms) on each day of the period. Then the following eleven estimates of the steepness of the curve were received:

1. Interest rate for 1 year – interest rate for 1 month.
2. Interest rate for 2 year – interest rate for 1 month.
3. Interest rate for 3 year – interest rate for 1 month.
4. Interest rate for 5 year – interest rate for 1 month.
5. Interest rate for 10 year – interest rate for 1 month.
6. Interest rate for 30 year – interest rate for 1 month.
7. Interest rate for 2 year – interest rate for 1 year.
8. Interest rate for 3 year – interest rate for 1 year.
9. Interest rate for 5 year – interest rate for 1 year.
10. Interest rate for 10 year – interest rate for 1 year.
11. Interest rate for 30 year – interest rate for 1 year.

In other words, the steepness of the curve should be estimated on the basis of one interest rate of the relatively short-term bond and one interest rate of relatively long-term bond. The difference between these interest rates shall be referred to as the steepness of the interest rate curve. Figure below reflects the historical data of the steepness of the interest rate curve. It is evident that, using exclusively daily data of the steepness of the interest rate curve, we may run into the following problems:

- Certain observations of interest rates curve steepness seem to be discrepant to neighbouring observations.
- The same value of the steepness of the interest rate curve may mean considerable steepness of the interest rate curve at one time and little steepness at other time.



Therefore, it seems to be useful do not rely on daily statistical data only and try to aggregate available information. In the present research, in addition to the daily values of the steepness of the curve, the following estimates were used:

1. Various moving averages.
2. Differences between interest rates and their moving averages.
3. Differences between different moving averages.

Therefore, 99 different estimates of steepness were received. The strength of relation of each of these estimates to the risk premiums of the following month was evaluated. The R^2 statistic was used as this relation strength measure. The table below shows the results of the evaluation:

Risk premium	Regressor estimator with the strongest relation to risk premium	R^2	R^2 average for all regressor estimators	Num. of data
One year versus one month	Difference between interest rates of 30 and 1 year bonds on the last day of the month minus one month average of this difference	0.05	0.005	216
Thirty years versus one month	Difference between interest rates of one year and one month bonds on the last day of the month	0.02	0.004	216

It is interesting to note that:

1. R^2 statistic for the best regressor of 1 year bond risk premium versus 1 month bond and 30 years bond risk premium versus 1 month bond is equal to 0.005.
2. R^2 statistic for the best regressor of 30 years bond risk premium versus 1 month bond and 1 year bond risk premium versus 1 month bond is equal to 0.003.

The relative stock market return

The stock market importance to the risk premiums of bonds may be described in the following way.

The level of risk premiums depends upon the risk tolerance of investors. If we recognize that risk tolerance is directly related to the level of welfare of the market participants, we may conclude that the higher (lower) standard of living (as compared to the past), the lower (higher) risk premiums may be required. In this research, the stock market performance data was used as the welfare proxy.

The estimation of the relative stock market return was based on the daily *Standard & Poor's 500* index data from January 3, 1950 up to February 19, 1998 (total number of items - 12 014)[10]. This index is referred to as a collection of stocks/shares of 500 companies in the USA. It enables to estimate the relative return from investment to stocks. As it was mentioned before, daily data of values cause irregular and extraordinary results. Therefore, in order to avoid misleading outcome, monthly average values were used. The indicator of risk tolerance was expressed by a variable inverse to the relative return of the stock market:

$$IND_j^w = \frac{(1-w)\cdot\sum_{i=1}^{\infty} MAVER_{j-i}\cdot w^{i-1}}{MAVER_j}$$

, where IND_j^w - inverse relative return of the stock market for month j; w - constant value $0 < w < 1$; $MAVER_j$ - monthly average of index value for month *j*. The following values of constant w were used: 0.9, 0.8,…,0.1, so nine possible regressors were received.

Each of these regressors was checked as regards the strength of their relation to risk premiums of the following month. The table below shows the results of this estimation:

Risk premium	Regressor estimator with the strongest relation to risk premium	R^2	R^2 average for all regressor estimators	Num. of data
One year versus one month	Inverse relative stock market return estimated when constant w is equal to 0.3	0.06	0.06	216
Thirty	Inverse relative stock market return	0.04	0.04	216

[10] The data was supplied by the Bloomberg agency.

years versus one month	estimated when constant w is equal to 0.5			

In addition:

1. R^2 statistic for the best regressor of 1 year bond risk premium versus 1 month bond and 30 years bond risk premium versus 1 month bond is equal to 0.04.
2. R^2 statistic for the best regressor of 30 years bond risk premium versus 1 month bond and 1 year bond risk premium versus 1 month bond is equal to 0.06.

Real interest rate

Real interest rate enables us to take into consideration two very important economic indicators, level of nominal interest rate and inflation. In order to estimate the real interest rate, the data of interest rate level and inflation are necessary. Historical data of inflation[11] used in the research is equal to the change of consumer prices over one year (e.g., level of inflation of March 1997 is equal to the change of prices from March 1996). In order to estimate of the interest rate level, interest rates of 1,2,3,5,10,30 were used. The latter were received during the evaluation of the steepness of the interest rate curve. Therefore, the estimate of real interest rate was based on these 30 more or less similar indicators (x=1,2,3,5,10,30):

- x year nominal interest rate on the last day of the month minus the most recent inflation level;
- monthly average of x year nominal interest rate minus the most recent inflation level;
- three months average of x year nominal interest rate minus the most recent inflation level;
- six months average of x year nominal interest rate minus the most recent inflation level;
- twelve months' average of x year nominal interest rate minus the most recent inflation level.

Each of these regressor estimators was checked as regards the strength of their relation to risk premiums of the following month. The table below shows the results of this estimation:

Risk premium	Regressor estimator with the strongest relation to risk premium	R^2	R^2 average for all regressor estimators	Num. of data
One year versus one month	3 months average of 3 years nominal interest rate minus the most recent inflation level	0.05	0.03	216
Thirty years	Monthly average of 5 years nominal interest rate minus the	0.02	0.02	216

[11] The data was received from the Bloomberg agency.

versus one month	most recent inflation level			

In addition:

1. R^2 statistic for the best regressor of 1 year bond risk premium versus 1 month bond and 30 years bond risk premium versus 1 month bond is equal to 0.02.
2. R^2 statistic for the best regressor of 30 years bond risk premium versus 1 month bond and 1 year bond risk premium versus 1 month bond is equal to 0.03.

Indicator of the market sentiment

From time to time, we can see that prices of bonds fluctuate along different direction and steepness trend. Investors try to use these trends and attempt to formulate methods of the construction of profitable investment strategies. Our research also will take into consideration this idea.

Therefore, interest rates of 1,2,3,5,10 and 30 years were selected[12]. Each of these indicators were expressed in 5 different ways: value on the last day of the month, monthly average, average of last three months, average of 6 last months and one year average. These values were used to estimate the market trends:

1. Interest rate on the last day of the month - monthly average of interest rate.
2. Interest rate on the last day of the month - 3 last months' average of interest rate.
3. Interest rate on the last day of the month - 6 last months' average of interest rate.
4. Interest rate on the last day of the month - last year average of interest rate.
5. Monthly average of interest rate – three last months' average of interest rate.
6. Monthly average of interest rate – six last months' average of interest rate.
7. monthly average of interest rate – last year average of interest rate

The sentiment indicator was computed using window equal to 0.05 (which resembles five b.p., i.e. 0.05 per cent):

$$indicator = \begin{cases} -1, \text{ if adequate indicator} > 0.05 \\ 0, \text{ if } -0.05 < \text{adequate indicator} < 0.05 \\ 1, \text{ if adequate indicator} < -0.05 \end{cases}$$

This indicator may be interpreted as:

[12] See the section on the steepness of the Interest Rate Curve.

- Equal to -1, when interest rate exceeds its historical average value by more than 0.05 (interest rate tend to increase / prices tend to decrease), and it shall be treated as a recommendation to sell securities.
- Equal to 1, when interest rate is less than its historical average by more than 0.05 (interest rate tend to decrease / prices tend to increase), and it shall be treated as a recommendation to purchase securities.
- Equal to 0, when no trends of interest rate fluctuation is observed, and it shall be treated as neutral recommendation.

Using the described method, 42 estimators of the sentiment indicator were received. Each of these estimators was checked as regards the strength of their relation to risk premiums of the following month. The table below shows the results of this estimation:

Risk premium	Regressor estimator with the strongest relation to risk premium	R^2	R^2 average for all regressor estimators	Num. of data
One year versus one month	Monthly average of 10 years interest rate minus its one year average	0.12	0.04	216
Thirty years versus one month	3 years interest rate on the last day of the month minus its one year average	0.19	0.05	216

In addition:

1. R^2 statistic for the best regressor of 1 year bond risk premium versus 1 month bond and 30 years bond risk premium versus 1 month bond is equal to 0.16.
2. R^2 statistic for the best regressor of 30 years bond risk premium versus 1 month bond and 1 year bond risk premium versus 1 month bond is equal to 0.12.

Combinations of Regressors

Taking into account the analysis carried out above the analysis of the best combinations of risk premiums' regressors was executed. The table below shows its results:

Risk premiums	Num. of regressors (decrease of residual variance)	Steepness of the Interest Rate Curve	Relative return of stock market	Real interest rate	Indicator of market sentiment	R^2	R^2 with respect to other risk premium
One year versus one month	1 (-)	-	-	-	+	**0.12**	0.16
	2 (8%)	-	+	-	+	**0.19**	0.20
	3 (2%)	-	+	+	+	**0.21**	0.20
	4 (1%)	+	+	+	+	**0.22**	0.21
Thirty years versus one month	1 (-)	-	-	-	+	0.19	**0.12**
	2 (5%)	-	+	-	+	0.23	**0.18**
	3 (1%)	-	+	+	+	0.24	**0.20**
	4 (0%)	+	+	+	+	0.24	**0.21**

First Conclusions

1. The market sentiment indicator yields to the highest R^2 statistic value.
2. R^2 statistic value for all regressors, with the exception of the market sentiment indicator, is higher in the case of risk premium of one year versus one month bond.
3. Thorough search of the best estimators of regressors was most successful and meaningful with regard to the steepness of the interest rate curve and the market sentiment indicator.
4. The sense of using different regressors' estimators with respect to each risk premium is most evident in the case of the steepness of the interest rate curve.
5. The analysis of selected combinations of regressors showed that only the increase of the number of regressors to two or three enables to notice the sensible increase of R^2 statistic value.

6. In both cases, the best combinations of regressors for any number of them coincide (only their estimators differ).
7. Using the combination 4 regressors, the values of R^2 in both cases are the same without regard to the selected regressors' estimators (the best for one of investigated risk premiums).

CONDITIONAL REGRESSION

Influence of the recent market performance

The theory indicates that the reaction of market participants to the losses and profit differs. Let us analyze the influence of such reaction to the risk premiums of bonds.
Let us assume that we try to predict the realizations of random process $Y = \{Y_t : t \in T\}$ using as regressors $X = \{(\chi_t^1, ..., \chi_t^n)' : t \in T\}$. Let us divide set T into two random subsets: $T_{H_Y^-} = \{t \in T : Y_{t-1} < 0\}$ and $T_{H_Y^+} = \{t \in T : Y_{t-1} \geq 0\} = T \setminus T_{H_Y^-}$. Therefore, we are able to divide each accidental process X, Y into two: $Y^- = \{Y_t : t \in T_{H_Y^-}\}$, $Y^+ = \{Y_t : t \in T_{H_Y^+}\}$, $X^- = \{(\chi_t^1, ..., \chi_t^n)' : t \in T_{H_Y^-}\}$ and $X^+ = \{(\chi_t^1, ..., \chi_t^n)' : t \in T_{H_Y^+}\}$ and try predicting Y^-, Y^+ using respectively X^-, X^+.
Therefore, the received realizations of process Y yields to the existence of realisations of sets $T_{H_Y^-}$ and $T_{H_Y^+}$, as well as Y^-, Y^+, X^-, X^+. These results enable us to use in following analysis traditional statistical techniques. Based on this analysis, one more research similar to described above was carried out.

Steepness of the interest rate curve

In case of the risk premium of one year versus one month bond, twice as high the value of R^2 was received. While in case of risk premium of 30 years versus one month bond, the increase was not as prominent. For all four cases, different best estimators of interest rate curve steepness were found.

Risk premium	Regressor estimator with the strongest relation to risk premium		R^2 (R^2 with respect to other risk premium)		R^2 average for all regressor estimators		Num. of data	
	-	+	-	+	-	+	-	+
One year versus one month	Monthly average of difference between 30 and 1 year interest rates minus two months average of this difference	Difference between 30-year and 1-month interest rates on the last day of the month minus monthly average of this difference	0.10 (0.02)	0.11 (0.01)	0.02	0.02	80	136
Thirty years versus one month	Difference between 3- and 1- year interest rates on the last day of the month minus 2 months average of this difference	Difference between 1-year and 1-month interest rates on the last day of the month	0.03 (0.03)	0.04 (0.03)	0.01	0.00	100	116

Relative Stock Market Return.

For both risk premiums, considerably increased values of R^2 were received. In the analysis above, the estimators of regressors were different for each risk premium as opposed to this analysis where they are identical.

Risk premium	Regressor estimator with the strongest relation to risk premium		R^2 (R^2 with respect to other risk premium)		R^2 average for all regressor estimators		Num. of data	
	-	+	-	+	-	+	-	+

One year versus one month	Inverse relative return of the stock market, computed with the constant w value equal to 0.1	Inverse relative return of the stock market, computed with the constant w value equal to 0.9	0.17 (0.10)	0.09 (0.07)	0.10	0.05	80	136
Thirty years versus one month	Inverse relative return of the stock market, computed with the constant w value equal to 0.1	Inverse relative return of the stock market, computed with the constant w value equal to 0.9	0.10 (0.17)	0.07 (0.09)	0.05	0.04	100	116

Real Interest Rate

As for this regression factor, its value did not increase as expected, with the exception of one case (see table below).

Risk premium	Regressor estimator with the strongest relation to risk premium		R^2 (R^2 with respect to other risk premium)		R^2 average for all regressor estimators		Num. of data	
	-	+	-	+	-	+	-	+
One year versus one month	Monthly average of 1 year interest rate minus the most recent inflation level	3 months average of 1 year interest rate minus the most recent inflation level	0.08 (0.03)	0.04 (0.01)	0.04	0.02	80	136

Thirty years versus one month	Monthly average of 5 years interest rate minus the most recent inflation level	3 months average of 1 year interest rate minus the most recent inflation level	0.03 (0.04)	0.01 (0.04)	0.03	0.01	100	136

Indicator of the Market Sentiment

In this case, considerable increase of R^2 values was noticed.

Risk premium	Regressor estimator with the strongest relation to risk premium		R^2 (R^2 with respect to other risk premium)		R^2 average for all regressor estimators		Num. of data	
	-	+	-	+	-	+	-	+
One year versus one month	Monthly average of 2 years interest rate minus 1 year average of this interest rate	5 years interest rate on the last day of month minus one year average of it	0.12 (010)	0.26 (0.26)	0.04	0.05	80	136
Thirty years versus one month	10 years interest rate on the last day of month minus one year average of it	5 years interest rate on the last day of month minus one year average of it	0.21 (0.03)	0.26 (0.26)	0.04	0.05	100	116

Combinations of Regressors

For both risk premiums, the increase of relation between the risk premiums and the steepness of the Interest Rate Curve was observed. In addition, it became evident that it is useful to employ four regressors in analysis.

Risk premiums		Num. of regressors (decrease of residual variance)	Steepness of the Interest Rate Curve	Relative return of stock market	Real interest rate	Indicator of market sentiment	R^2	R^2 With respect to other risk premium
-	One year versus one month	1 (-)	-	+	-	-	**0.17**	0.11
		2 (11%)	+	+	-	-	**0.25**	0.16
		3 (8%)	+	+	+	-	**0.31**	0.18
		4 (6%)	+	+	+	+	**0.36**	0.20
-	Thirty years versus one month	1 (-)	-	-	-	+	0.21	**0.16**
		2 (15%)	-	+	-	+	0.32	**0.25**
		3 (3%)	-	+	+	+	0.34	**0.28**
		4 (4%)	+	+	+	+	0.37	**0.30**
+	One year versus one month	1 (-)	-	-	-	+	**0.26**	0.27
		2 (11%)	+	-	-	+	**0.34**	0.30
		3 (5%)	+	-	+	+	**0.37**	0.31
		4 (3%)	+	+	+	+	**0.39**	0.31
+	Thirty years versus one	1 (-)	-	-	-	+	0.26	**0.20**
		2 (3%)	-	+	-	+	0.28	**0.25**
		3 (1%)	-	+	+	+	0.29	**0.27**

		month	4 (0%)	+	+	+	+	0.29	**0.27**

Due to the fact that $Y1b1_t = \begin{cases} Y1b1_t^{-}, \text{kai } t \in T_{T_Y^{-}} \\ Y1b1_t^{+}, \text{kai } t \in T_{T_Y^{+}} \end{cases}$, the table below summarize the results of application of conditional regression.

Risk premium	Num. of regressors (decrease of residual variance)	R^2	Former R^2	R2 difference	R2 difference (per cents)
One year versus one month	1 (-)	0.21	0.12	0.09	75%
	2 (11%)	0.30	0.19	0.11	58%
	3 (6%)	0.34	0.21	0.13	62%
	4 (6%)	0.38	0.22	0.16	73%
Thirty years versus one month	1 (-)	0.25	0.19	0.06	32%
	2 (9%)	0.32	0.23	0.09	39%
	3 (1%)	0.33	0.24	0.09	38%
	4 (1%)	0.34	0.24	0.10	42%

Therefore, upon application of conditional regression, quite considerable increase of R^2 was received (e.g. using the combination of four regressors, it reached 73% for one year bond risk premium versus one month bond and 42% for 30 years bond risk premium versus one month bond). However, before we can apply the results of this research in practice, it is necessary to check the stability of these results.

Up to the present moment, in order to forecast risk premiums, all available data was used. E.g. we predicted random process $Y = \{Y_t : t \in T\}$ by linear regression of random variables $X = \{(x_t^1, ..., x_t^n)' : t \in T\}$, i.e. $\hat{Y}_t = x'_t \cdot b$, where $\hat{Y}_t$ is defined as the forecast of Y_t, $t \in T$, and vector b is estimated using the ordinary least squares method all available realizations of Y_t, x_t, ($t=1,\ldots,t_{max}$). Now, let us analyze the possible shift of results in case vector b is estimated using exclusively the data available until time t. In such situation, the forecast of process Y_t is equal to:

$\hat{Y}_t = x'_t \cdot b_t$, where b_t is estimated using the ordinary least squares method and all available realizations of Y_j, x_j, where $j=1,\ldots,t-1$.

The results of the analysis (see the table below) confirm the assumption that conditional regression is more suitable for the forecast of risk premiums as compared to ordinary linear regression. The results of R^2 improved by 46 per cent for one year bond risk premium versus one month bond and 21 per cent for 30 years bond risk premium versus one month bond. Besides, both methods of forecast turned to be more reliable than random walk. In addition to that, the possibility to predict negative or positive sign of premiums was evaluated because it is important in order to make decisions regarding the investment strategies. In this respect, random walk turned to be worthless as compared to the two methods of consideration, as well, Even though using conditional regression it was a little harder to predict the sign of difference between returns of 1 year and 1 month bonds.

Risk premium	R^2 (probability that the correlation of the predicted and actual value of the risk premium is equal to 0)			Percentage of correct sign predictions		
	Conditional regression	Linear regression	Random walk	Conditional regression	Linear regression	Random walk
One year versus one month	0.19 (0.0001)	0.13 (0.0001)	0.06 (0.0021)	69	74	63
Thirty years	0.23	0.19	0.02	70	61	60

versus one month	(0.0001)	(0.0001)	(0.0933)			

IV. APPLICATION OF RECEIVED RESULTS IN PRACTICE OF INVESTMENT

In order to check if the received results of the risk premium forecasts are precise enough to be applied to actual investments, let us discuss the following situation:

Let us assume that, on the last day of every month, investor is able to invest to 1-month-, 1-year- and 30-year- term bonds. After one month, the existing investment portfolio is priced at market price, and the owner may sell the bonds and repurchase them in other proportions. Within the framework of this situation, the following passive strategies of investment were analyzed[13]:

1. Every month, the investor invests all his funds into 1-month bond (the strategy with the smallest level of risk).
2. Every month, the investor invests all his funds into 1-year bond.
3. Every month, the person invests all his funds into 30 year bond (the strategy with the biggest level of risk).
4. Every month, the investor distributes his funds equally among 1 month, 1 year and 30 year bonds.

If we denote the volume of investment into 1-month, 1-year and 30-year bonds as W_i, (i=1,2,3), respectively, strategies 1 to 4 may be put in the following form:

1. $W_1=1$, $W_2=W_3=0$
2. $W_1=0$, $W_2=1$, $W_3=0$
3. $W_1=W_2=0$, $W_3=1$
4. $W_1=W_2=W_3=1/3$

Taking into consideration the forecasts of risk premiums, we may formulate alternative strategies to each of the passive strategy above using the following formulas (this method, of course, does not claim to be the best way to combine predictions in order to construct investment portfolio. The aim is just to look how it could be done):

$$u_1 = w_1 - k \cdot \hat{p}_{y1b1} - k \cdot \hat{p}_{y30b1},$$

$$u_2 = w_2 + 2 \cdot k \cdot \hat{p}_{y1b1} - k \cdot \hat{p}_{y30b1},$$

[13] For the purpose of this research, passive strategies are realized as strategies not supported by forecasts (future predictions). However, it does not mean that the investments may not be effected by other means

$$u_3 = w_3 - k \cdot \hat{p}_{y1b1} + 2 \cdot k \cdot \hat{p}_{y30b1},$$

$$v_i = u_i + \min(u_1, u_2, u_3, 0), \qquad (3)$$

$$\hat{w}_i = v_i / (v_1 + v_2 + v_3),$$

i=1,2,3, $w_i -$ weights of investments according to the passive strategies, $\hat{w}_i -$ weights of investments according to the alternative active strategies; $\hat{p}_{y1b1}$, $\hat{p}_{y30b1}$ - forecasts of risk premiums of 30 and 1 year bonds with respect to 1 month bond, respectively, k [14]-“aggressiveness” constant.

Therefore, there are four passive strategies of investment and four active strategies directly oriented to the passive strategies[15]. In the analysis of the attractiveness of these strategies from the retrospective point of view, the data of the period from July, 1984 to January, 1998 were used because it was the period covered by the forecasts of the risk premiums. Table 1 includes the statistical data for the passive strategies. It enables to conclude that the higher the risk of investment, the more evident possibility of larger profits. However, it involves the wider amplitude of fluctuation of the value of investments as well as the value of variation coefficient[16].

No.	Passive strategy	Average return per month (%)[17]	Minimal return per month (%)	Maximal return per month (%)	Variation coefficient	Num. of months with positive return (%)
1	W_1=1, W_2=0, W_3=0	5.35	2.36	10.07	31	100
2	W_1=0, W_2=1, W_3=0	7.02	-1.22	23.51	59	96
3	W_1=0, W_2=0, W_3=1	12.59	-70.39	126.08	307	64
4	W_1=W_2=W_3=1/3	8.97	-32.98	66.12	217	71

[14] Further in the research k=10.
[15] Each of them differs from passive strategies in proportion to forecasts of risk premiums.
[16] In practice, the attractiveness of investment strategies is evaluated by the variable that is inverse variation coefficient, without regard to its evident shortcomings.
[17] All return is annualized: Annual return per month is equal to 12*100*(value at the end of the month - value at the beginning of the month) / value at the beginning of the month

Table 1.

Then, for each passive strategy, an alternative strategy was estimated using formulas presented above. As it was mentioned before, the purpose of the alternative strategies was to modify passive strategies of investment with regard to the forecasts of the risk premiums and the value of constant k. Table 2 presents the results of the historical analysis of 4 active strategies. First, it is evident that the application of relative regression enabled to achieve higher returns, at the same time risk factors of the strategies decreased. This conclusion proves the necessity of the application of the relative regression in practice. In all cases, compared to the respective passive strategies, the active strategies enabled to improve the indicator of average return, and to decrease risk factors, as in the last two cases.

No.	Alternative strategy corresponding to following passive strategy	Average return per month (%)[1)]	Minimal return per month (%)[1)]	Maximal return per month (%)[1)]	Variation coefficient[1)]	Num. of months with positive return (%)[1)]
1.1.	W_1=1, W_2=0, W_3=0	9.09 (8.53)	-11.26 (-13.77)	53.29 (54.14)	100 (106)	98 (94)
1.2.	W_1=0, W_2=1, W_3=0	10.30 (9.74)	-10.75 (-16.05)	51.71 (53.46)	100 (107)	94 (92)
1.3.	W_1=0, W_2=0, W_3=1	14.67 (14.25)	-55.45 (-67.02)	122.60 (121.49)	218 (226)	66 (66)
1.4.	W_1=W_2=W_3=1/3	13.56 (12.79)	-26.40 (-33.11)	87.63 (80.67)	135 (144)	90 (89)

Table 2. [1] The first number corresponds to conditional regression and the second one to ordinary linear regression.

Table 3 presents more information necessary for detailed comparison of passive and active strategies. It shows that we failed to achieve the expected result in the case of the most risky strategy (Line 3). More favorable results of the remaining three strategies allow us to believe that the risky passive strategy may be overcome by using more aggressive constant k (see formula (3)). Except for the most risky strategy, the passive strategies were behind the active ones in 62-71 per cent of all months. Moreover, with high level of confidence, we may deny the hypothesis that the average return of the active strategies not differs from the return of the passive strategies.

No.	Alternative strategy minus respective passive strategy	Average return per month (%)[1)]	Minimal return per month (%)[1)]	Maximal return per month (%)[1)]	Variation coefficient[1)]	No. of months with positive return (%)[1)]	Probability that the difference between average monthly return is equal to 0[1)]
1.1.-1	W_1=1, W_2=0, W_3=0	3.74 (3.18)	-15.73 (-19.43)	43.71 (44.56)	231 (274)	62 (63)	0.0001 (0.0001)
1.2.-2	W_1=0, W_2=1, W_3=0	3.28 (2.72)	-15.75 (-18.97)	39.03 (36.26)	239 (288)	65 (55)	0.0001 (0.0001)
1.3.-3	W_1=0, W_2=0, W_3=1	2.08 (1.66)	-28.00 (-27.94)	64.70 (47.55)	518 (596)	44 (44)	0.0147 (0.0338)
1.4.-4	W_1=W_2=W_3= 1/3	5.24 (4.48)	-28.00 (-27.94)	44.93 (40.58)	194 (231)	71 (61)	0.0001 (0.0001)

Table 3. [1] The first number corresponds to conditional regression and the second one to ordinary linear regression.

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