

# Why do we teach the mathematics that we do? The case of Lithuanian school mathematics

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**Abstract.** Due to the changes of education system the school mathematics in Lithuania have acquired the elements of commercial-administrative mathematics of ancient times. Among other consequences the opportunities of school children to achieve the standards of mathematical reasoning are limited.

**Keywords:** learning paradigm, teaching paradigm, education reforms, school mathematics, political economy of math education.

## Introduction

During the last decades basic educational attitudes in Lithuania could be described in terms of traditional vs. progressive education. Even though this terminology is not used by education scientists in Lithuania. It should also be noted that traditional education in this country is linked to the heritage of soviet education.

In this paper we aim to justify the statement that the learning and teaching paradigms have caused school mathematics to be taught in a way which does not foster reasoning skills and in turn does not prepare the learners for future challenges. To achieve this we will use:

- 1 The theoretical framework of H. Harouni [7] for understanding the content and pedagogy of school mathematics as a set of practices reflecting socio-political values.
- 2 The content analysis of most popular mathematics textbooks of the last 40 years in Lithuania.
- 3 The evaluation of reforms of school education in Lithuania during the last decades.

## 1 Political economy of mathematics education

In [7] Houman Harouni asks: “Why do schools teach the mathematics that they do?” He argues that the justification offered by national education systems are not

convincing. We too the teachers, when asked by students “Where do we use this math in everyday life?”, have no satisfactory answers.

H. Harouni looks at the history of mathematics education with roots in western culture spanning about 5000 years to find an answer to his own question. He traces the origins of modern math education to the early institutions in which mathematics served a clear utilitarian purpose. He finds common and unexamined assumptions regarding the place and form of mathematics education in our society. The answer is given in the form of a theoretical framework, which he describes in four principles [7, p. 71]:

- “The economic purpose of math defines its most basic characteristics.
- The economic characteristics of math impact how it can be taught.
- The institutional setting within which math is taught also modifies the character of its practice.
- All of the above aspects impact one another in relation to the socio-economic forces that shape them.”

To justify our claim about school mathematics in Lithuania we follow the third principle and show how our national education system has caused a change during the last 30 years.

Harouni showed that there is no a single “basic” mathematical practice. Instead he specifies four categories of school mathematics:

- commercial-administrative mathematics;
- philosophical mathematics;
- artisanal mathematics;
- social-analytic mathematics.

We describe features of school mathematics from the first two categories since only these are used in the next section.

A school mathematics is classified as commercial-administrative provided it helps to forecast outcomes of a suitable economic activity unambiguously. For example, math should provide unique answers to the question “ $12 + 15 = ?$ ” In more detail the basic features of school mathematics from the first category are:

- main attention is devoted to computation;
- arithmetic operations are always presented in the same order (addition, subtraction, multiplication, division);
- a correct execution of a procedure for making arithmetic operations;
- no or very little attention is given to formulating underlying principles behind taught procedures.

A school mathematics is classified as philosophical if it has features of ancient Greek mathematics. A typical question of this kind of mathematics is “ $27 = ?$ ” In more detail the basic features of school mathematics from the second category are:

- a lot of attention devoted to pattern seeking;
- a search for meaning (by defining concepts);
- use of proof to justify facts;

- word puzzles in place of word problems in a real life context.

The features of philosophical mathematics agree with basic elements of mathematical reasoning.

We will show in the next section how elements of philosophical mathematics that were present in math textbooks in 1980s have been lost in the latest textbooks and were replaced by clear examples of commercial-administrative mathematics.

## 2 The content analysis of mathematics textbooks

By comparing the content of the current most popular mathematics textbook with the content of older ones we show that the elements of mathematical reasoning have disappeared gradually. Specifically we compare the textbooks for the eighth grade published in 1982, 1999 and 2013. We will illustrate the changes by showing how the topic “powers with integer exponents” is presented in each of these textbooks.

Let us look at the textbook published in 1982. The topic begins with the phrase: „What is the **meaning** of the expression  $10^{-24}$ ?” Note that the textbook intends to explain the meaning of the power. This is an element of mathematics which belongs to the philosophical category. Further in the textbook it is explained that the power with the negative exponent is a notation for the expression whose meaning was defined earlier. As usual in mathematics, the notation is presented as a **definition** of a concept with known implications.

The next section of the same textbook published in 1982 describes properties of powers with integer exponents. The **proof** of properties is illustrated for powers with specific numbers as exponents but with a general base. It is written that a proof for the general case is not given here. Moreover, it is shown that if one wants to have these properties then the form of the definition stated earlier is necessary. In this way the topic in the textbook is presented so that it has the basic elements of mathematical reasoning.

Now we look at the textbook published in 1999. No mention about the meaning of the power with integer exponent. However the properties of powers with positive integer exponents for specific numbers are used to motivate the **agreement**:

$$a^{-n} = \frac{1}{a^n} \quad (a \neq 0).$$

Here and in other parts of the textbook no word “definition” is used.

The properties of powers with integer exponents are described in the next section of the same textbook published in 1999. However no proofs are mentioned. Instead, the properties are **illustrated** with concrete numbers before formulating in a general case. The examples are stated so that they look as if they justify the properties.

Finally we look at the textbook published in 2013. The topic “powers with integer exponents” starts with the question: “How to **compute** the value of the power with negative integer exponent?” In this case the focus is on a computation and no talk about meaning or even about agreement is presented. The answer to the above question is given by the formula:

$$a^{-n} = \frac{1}{a^n}, \quad a^0 = 1 \quad (a \neq 0).$$

Further in the book the formula is discussed when  $a = 2$ . It is suggested to note that if the value of the positive integer exponent is reduced by one then the value of the power becomes twice less:

$$2^3 = 8, \quad 2^2 = 4, \quad 2^1 = 2.$$

Further in the textbook it is written: therefore

- the value of the power  $2^0$  is obtained once  $2^1$  is divided by 2;
- the value of the power  $2^{-1}$  is obtained once  $2^0$  is divided by 2;
- the value of the power  $2^{-2}$  is obtained once  $2^{-1}$  is divided by 2.

What in the textbook published in 1982 was used as an indication has now become a justification of a given formula. Such a justification may be interpreted as giving the same logical status for the property

$$a^{m-1} = \frac{a^m}{a}$$

with  $m = 2, 3, \dots$  and with  $m = 1, 0, -1, \dots$

In the next section of the same textbook published in 2013 the properties of the powers with integers exponents are formulated with a single statement added: “The properties of powers with positive integer exponents also **apply** for the powers with integer exponents”.

Furthermore, the textbook published in 2013 contains hundreds of exercises. Due to our estimation, to execute them one needs to know a single procedure. Some exercises are stated as “word problems” in a real life context. The textbook has about 800 exercises and only 8 of them require some thinking. Since the described textbook is currently the most popular textbook of mathematics in Lithuania, the lack of explanation, the focus on computation and the character of exercises allow us to claim that current school mathematics belongs to the category of administrative-commercial mathematics.

### 3 The reform of educational content and its consequences to school mathematics

We shall suggest an explanation why school mathematics in Lithuania acquired the elements of commercial-administrative mathematics. According to the theoretical framework given by H. Harouni it may have been caused by the changes in the institutional setting of national education system.

The reform of the education system of Lithuania has been carried out in several stages: stage 1 was implemented from 1988 until 1997; stage 2 from 1998 until 2002; stage 3 from 2003 and is to be complete in 2022. Initially the time period of the 3rd stage was planned until 2012. Also, the final stage termed as the strategy rather than the reform. The reform of educational content (programs, textbooks, teaching methods, teacher preparation programs) was one of its several aspects. It have been driven by confronting views at what is the essence of learning and teaching. The two extreme points of view are the learning paradigm and the teaching paradigm (see e.g. [1], [8, pp. 6–14]). The first one is a guiding principle to be established into the

education system and the second one is teaching paradigm describing what should be abolished from the education system (see e.g. [2]).

Briefly, the learning paradigm is a belief that the most efficient way to teach children is to allow them to develop their own knowledge, understanding, abilities and values using the surrounding environment. In this case a teacher is a helpmate, a mentor and a creator of learning environment. The aims of learning are formulated by students, while teachers help them to relate these aims with the curriculum [12, p. 29].

Implementation of the learning paradigm is the guiding direction of the reform of the education system approved by the decision of the Minister [5, Section 9]. The learning paradigm repeats the ideas of the child-centered paradigm in a progressive pedagogy which became a prominent educational philosophy at the beginning of the 20th century in the United States. The learning paradigm as well as progressivism have difficult relations with knowledge. However, as evidence shows, often the most efficient way of gaining knowledge for novices is through guided instruction [9].

According to the reformers of the education system, the teaching paradigm is a transfer of theoretical knowledge seeking specific results, for example, final teaching achievements. Teaching is an imposition on a child from the outside, contradicting to learning from the inside. A teacher in this case is a transferor of knowledge framing the aims of learning. While the learner is merely a passive recipient of information [12, p. 29].

Based on confrontation between learning and teaching the current ideology permeated through the education system including didactics and curriculum. Different learning theories are used to justify and implement changes. The ideas of constructivism are used to transform didactics according to the learning paradigm [1, p. 50]. While teaching paradigm is explained using behaviorism which has a negative connotation in the current ideology [1, p. 52]. Lithuanian education ideology is in disagreement with the theory of instructional design which suggests that there is no single best approach to teaching and learning [6, p. 60].

As consequence of the child-centered ideology, the National Curriculum Framework is based on developing competencies. Competences are claimed to be a whole of knowledge, skills, attitudes in a certain field, and proven ability to fulfill tasks. In fact, according to the reformers: “When developing a modern curriculum, it is important to stress not the input (what to teach) but the expected outcome, i.e. what skills and attitudes students should to acquire and what competences to develop in order to successfully build up their personal life and integration into modern society” [10]. So, it follows that the educational content can be simplified if it helps to seek competencies. Even more: “the scientific logic of disciplines should be abandoned for the sake of child-centered learning logic” [10, p. 4]. Officially, previous orientation to knowledge is considered as weakness of the Lithuanian educational system [4, Section 9.1].

These comments explain why school mathematics in Lithuania acquired the elements of commercial administrative mathematics. Additional evidence can provide the works [11] and [3] describing changes in mathematics didactics related to implementation of the learning paradigm. These works confirm that what we see in current mathematics textbooks is a consequence of the reform of educational content. In particular, it is claimed [3, p. 166] that mathematics teachers use textbooks as a primary source for planning their lessons.

## 4 Conclusion

We have shown that three decades long reform of Lithuanian education system resulted in prominence child-centered paradigm and general skill based curriculum. At the same time, knowledge and teacher-led instruction have been given a bad reputation. As a result, adequate teaching of mathematical reasoning is not available to most school-children in the current education system.

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## REZIUMĖ

### Kodėl mokome tokios matematikos, kokios mokome?

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Švietimo sistemos kaitos dėka mokyklinė matematika Lietuvoje įgijo senųjų laikų komercinės-administracinės matematikos bruožus. Be kitų svarbių pasekmių, toks mokyklinės matematikos pobūdis reiškia, kad mūsų mokinių tinkamas matematinio samprotavimo ugdymas yra ribotas.

*Raktiniai žodžiai:* mokymosi paradigma, mokymo paradigma, švietimo reforma, mokyklinė matematika, matematikos mokymo politinė ekonomika.