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MODELING AND FORECASTING EXCHANGE RATES

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Abstract. This paper investigates models for the euro exchange rate against the currencies of Denmark, Poland, the United States, and the United Kingdom. The objective of this paper is to compare different methods of modeling and out-of-sample forecasting. One of the techniques is cointegration relation, which is implemented through a vector error correction model. The existence of cointegration supports the long-run relationship between the nominal exchange rate and a number of fundamental variables. The evidence presented in this paper shows that a simple multivariate random walk model tends to have superior predictive performance, compared to other exchange rate models, for a period of less than one year.

Key words: exchange rate, random walk, vector error correction model, forecast.

1. Introduction

Globalization has been one of the most important trends in the world economy in the recent decades, and the exchange rate is one of the factors that has a tremendous ascendancy. Consequently, the exchange rate affects any economy through many channels. It has a direct impact on the nation's international trade, economic growth, capital flows, interest rates and even inflation. Economic theories state that the exchange rate is determined by fundamental variables listed above, but since the start of the floating exchange rate regime in the early seventies, an increasing amount of empirical anomalies seem to contradict the existing exchange rate theories. Thus, in this paper, the link between exchange rates and fundamentals is analyzed. The aim of this paper is not to attempt to develop or estimate a particular exchange rate model. Instead, an analysis is made using existing and conventional class of economic models, in which the exchange rate is determined by the current and expected future values of observable fundamentals and unobservable shocks. However, the uniqueness of this paper is in the choice of exchange rates, investigated time period and multivariate framework. So, the paper is organized as follows: the theoretical background and rationale for all the variables is discussed in Section 2, Section 3 presents the data and the estimation methodology, Section 4 highlights the results, in Section 5 conclusions are made.

2. Literature review

In spite of the considerable amount of work done by both academic and policy researchers in analysing the sources of exchange rates, especially after the breakdown of the Bretton Woods system in March 1973, modeling the exchange rate remains a challenge. This is partly reflected in the existence of numerous theoretical models of exchange rate determination and several modeling approaches. Before the breakdown of Bretton Woods, most of the exchange rate models were based on the fixed price assumptions. The basic models of exchange rate determination are known as purchasing power parity (PPP) and uncovered interest parity (UIP). The former states that the relative price of two identical, domestic and foreign, baskets of goods is constant when expressed in a common currency, while the latter states that the difference in interest rates between two countries is equal to the expected change in exchange rates between the countries' currencies. However, for the post-Bretton Woods floating period, Meese and Rogoff (1983) highlighted poor out-of-sample forecasting performance of various univariate exchange rate models such as the relative PPP or monetary model. Specifically, they showed that the post-sample forecasts of foreign exchange rates among major countries are splintered by a random walk, especially in the short-term, that does not use any fundamentals for the forecasting period. However, empirical evidence shows that the fundamentals may contain predictive power for exchange rate movements in the long-term. More precisely, Chinn and Meese (1995) used different techniques to estimate fundamental exchange rate models, which showed poor forecasting performance for the short-term and better results in the long-term forecasting performance. Dua and Ranjan (2014) analyze several univariate exchange rate models, which are random walk, monetary model and various extensions thereof in the vector autoregressive (VAR) and Bayesian VAR framework. They results showed that both VAR and BVAR generally outperform the random walk based on forecasts in a projection of one year and less than one year. Ghalayini (2014) estimated several univariate ARIMA models using lagged values as explanatory variables and extensions with two fundamental explanatory variables, which showed that the PPP theory explains the main part of the euro/dollar exchange rate, and in this case the interest differential of the fundamental variable explains only a small proportion of subsequent changes in exchange rates. Zhang, Lowinger and Tang (2007) investigated the monetary model by applying the multivariate time series model. They showed that the existence of cointegration between the exchange rates and various fundamental variables supports the monetary model as a long-term relationship. Moreover, the estimated VEC model outperforms the random walk in the out-of-sample forecast for a time horizon of less than 12 months. Carriero, Kapetanios and Marcellino (2008) proposed a Bayesian VAR with normal-inverse Wishart prior to forecast panel exchange rates, which was preeminent to the random walk. Empirical analyzes differ in their choices of underlying exchange rate fundamentals, but all of them showed a link between the exchange rates and fundamentals in the long-term period.

Simple extensions of the traditional VAR models, which incorporate volatilities as explanatory variables, make an improvement in the specification and formulation of uncertainty for exchanges rates. Polasek and Ren (1999) proposed the multivariate VAR-GARCH-M model for the exchange rates, which outperforms VAR. Another example is Balter, Dumitrescu and Hansen (2015), where they used a realized GARCH model and showed that it outperforms the traditional CCC, cDCC and diagonal BEKK models up to 10 period-ahead.

Although a random walk without drift has proven to be a very competitive model in forecasting exchange rates, there is an evidence that there are models which can outperform the random walk.

3. Methodology and data

The purpose of this study is two-fold. First, we want to establish the empirical validity of the exchange rate models determination. Second, we want to determine whether the forecasts based on the exchange rate models outperform the multivariate "random walk". Let us denote it as a Naive model. The cointegration, vector error correction model (VECM) and GARCH methods are utilized in this paper to examine the relationships between the exchange rate and various economic fundamental variables.

The countries included in this study are European Monetary Union (EMU), Denmark, Poland, the United Kingdom, and the United States. These countries have a floating exchange rate regime, in which a currency's value is allowed to fluctuate in response to foreign-exchange market mechanisms.

The data used in the analysis are obtained from Eurostat[1]. The time horizon chosen is from January 1995 to March 2016. The data used in the modeling are as follows:
e_t : Market rate of a foreign currency per euro, in logarithm;
i_t : Short-term money market interest rate (3-month);
m_t: Seasonally adjusted M1 in national currency, in logarithm;
y_t: Industrial production index, in logarithm.

The EMU was chosen as the home country, while Denmark, Poland, the United Kingdom, and the United States were selected to represent foreign countries. Euro was selected as the base currency. Therefore, four pairs of countries, namely, EUR/DKK, EUR/PLN, EUR/GBP, and EUR/USD were chosen to test the relationship suggested by the exchange rate models.

[1] http://ec.europa.eu/eurostat/data/database

Let us consider four exchange rate models.

1. **Naive model [8]**

   Suppose e_t is a $n \times 1$ matrix of exchange rates. In a univariate case, a random walk is an AR(1) process, where $\rho = 1$:
   $e_t = e_{t-1} + \varepsilon_t$.
   Clearly, the series is not stationary. In order to make the time series stationary, one needs to take its first order difference
   $\triangle e_t = e_t - e_{t-1} = \mu + \varepsilon_t$.
   Thus, the random walk is said to be integrated of order one, or I(1). If the first difference follows the random walk, again take the difference of the first difference (integrated of order two) and so on. In order to estimate multivariate time series, the vector autoregressive model (VAR) will be used.

2. **Vector error correction model (VECM) [6]**

   An n-dimensional time series is cointegrated if some linear combination of the component variables is stationary. If cointegration has been detected between series, we know that there is a long-term relationship between them, so we apply the vector error correction model (VECM) in order to evaluate the short-term properties of the cointegrated series, i.e. VECM adjusts to both short-term changes in variables and deviations from the equilibrium. A negative and significant coefficient of the error correction term (ECT) indicates that any short-term fluctuations between the independent and the dependant variable will give rise to a stable long-term relationship between variables. The regression equation form of the VECM is as follows:

   $$\triangle e_t = \delta + \Pi e_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \triangle e_{t-i} + \varepsilon_t,$$

   where
   e_t: $n \times 1$ the exchange rates matrix,
   $\Pi = \alpha\beta$: α is $n \times r$ and β is $r \times n$ matrices of the error correction term (denotes as ect1 in the tables below). The cointegration rank r shows the number of cointegrating vectors. For instance, a rank of two indicates that two linearly independent combinations of non-stationary variables will be stationary,
   Γ_i: $n \times n$ the short-term coefficient matrix. It tells about the rate at which the previous period's disequilibrium of the system is being corrected,
   ε_t: $n \times 1$ vector of iid errors.

3. **VECM with fundamentals (flexible price monetary model) [3;12]**

   The VECM with fundamentals model is described as above, but with additional exogenous variables: $\triangle e_t = \delta + \Pi e_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \triangle e_{t-i} + \alpha(i_t - i_t^*) + \beta(e_t - e_t^*) + \kappa(m_t - m_t^*) + \theta(y_t - y_t^*) + \varepsilon_t$,
   where an asterisk denotes the foreign value of the object, e_t is the exchange rate, i_t is the short-term interest rate, m_t is M1 and y_t is the industrial production index.

4. **DCC-GARCH model [9]**

   The dynamic conditional correlation (DCC-) GARCH is a class of multivariate GARCH estimators where the covariance matrix, $\Sigma_{t|t-1}$, is decomposed into conditional standard deviations, $\boldsymbol{D_t}$, and a correlation matrix, $\boldsymbol{R_t}$. In the DCC-GARCH model, both $\boldsymbol{D_t}$ and $\boldsymbol{R_t}$ are designed to be time varying.

   Suppose we have exchange rate returns, $\triangle e_t$, of n currencies with the expected value 0 and the covariance matrix $\Sigma_{t|t-1}$. Then, the Dynamic Conditional Correlation (DCC-) GARCH model is defined as:

$$\triangle e_t = \mu_t + a_t,$$

$$a_t = \Sigma^{\frac{1}{2}}_{t|t-1} z_t,$$

$$\Sigma_{t|t-1} = D_t R_t D_t,$$

where
$\triangle e_t$: $n \times 1$ vector of returns of n currencies at time t,
a_t: $n \times 1$ vector of mean-corrected returns of n currencies at time t, i.e. $\mathbb{E}a_t = 0$, $Cov(a_t) = \Sigma_{t|t-1}$,
μ_t: $n \times 1$ vector of the expected value of the conditional $\triangle e_t$,
$\Sigma_{t|t-1}$: $n \times n$ matrix of conditional variances a_t at time t,
$\Sigma^{\frac{1}{2}}_{t|t-1}$: any $n \times n$ matrix at time t such that $\Sigma_{t|t-1}$ is the conditional variance matrix of a_t. $\Sigma^{\frac{1}{2}}_{t|t-1}$ might be obtained by a Cholesky factorization of $\Sigma_{t|t-1}$,
D_t: $n \times n$, diagonal matrix of conditional standard deviations of a_t at time t,
R_t: $n \times n$ conditional correlation matrix of a_t a time t,
z_t: $n \times 1$ vector of iid errors such that $\mathbb{E}(z_t) = 0$ and $\mathbb{E}(z_t z_t^T) = I$.

The elements in the diagonal matrix D_t are standard deviations from univariate GARCH models, i.e. $D_t = diag(\sqrt{\sigma_{1t,t|t-1}}, \ldots, \sqrt{\sigma_{nt,t|t-1}})$ where $\sigma_{it,t|t-1} = \alpha_{i0} + \sum_{q=1}^{Q_i} \alpha_{iq} a^2_{i,t-q} + \sum_{p=1}^{P_i} \beta_{ip} \sigma_{i,t-p}$.
R_t is the conditional correlation matrix of the standardized disturbances ε_t, i.e. $\varepsilon_t = D_t^{-1} a_t \sim \mathcal{N}(0, R_t)$. Since R_t is symmetric, the elements of $\Sigma_{t|t-1} = D_t R_t D_t$ are $[\Sigma_{t|t-1}]_{ij} = \sqrt{\sigma_{it,t|t-1}\sigma_{jt,t|t-1}}\rho_{ij}$ where $\rho_{ii} = 1$. Requirements: 1) $\Sigma_{t|t-1}$ has to be positive definite because it is a covariance matrix. To ensure that $\Sigma_{t|t-1}$ is positive definite, R_t has to be positive definite; 2) by definition, all elements in the correlation matrix R_t have to be equal to or less than one. To ensure both of these requirements in the DCC-GARCH model, R_t is decomposed into:

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1}$$

$$Q_t = (1 - a - b)\bar{Q} + a\varepsilon_{t-1}\varepsilon^T_{t-1} + bQ_{t-1},$$

where $\bar{Q} = Cov(\varepsilon_t \varepsilon_t^T)$ is the unconditional covariance matrix of the standardized errors ε_t. $\bar{Q}$ can be estimated as $\bar{Q} = \frac{1}{T}\sum_{t=1}^{T} \varepsilon_t \varepsilon_t^T$. The parameters a and b are scalars, Q_t^* is a diagonal matrix with the square root of the diagonal elements of Q_t at the diagonal $Q_t = diag(\sqrt{q_{11t}}, \ldots, \sqrt{q_{nnt}})$. Q_t^* rescales the elements of Q_t to ensure the second requirement: $|\rho_{ij}| = \frac{|q_{ijt}|}{|q_{iit} q_{jjt}|} \leq 1$. Further, Q_t has to be positive definite to ensure that R_t is positive definite. Conditions $a \leq 0$, $b \leq 0$ and $a + b \leq 1$ is a guarantee that $\Sigma_{t|t-1}$ is positive definite.
The correlation structure can be extended to the general DCC(M,N)-GARCH model:

$$Q_t = (1 - \sum_{m=1}^{M} a_m - \sum_{n=1}^{N} b_n)\bar{Q}_t + \sum_{m=1}^{M} a_m \varepsilon_{t-1}\varepsilon^T_{t-1} + \sum_{n=1}^{N} b_n Q_{t-1}.$$

In this paper, only the DCC(1,1)-GARCH model will be studied.

In order to evaluate forecast performance against other models, out-of-sample validation will be performed. The models are initially estimated using monthly data over the period from January 1995 to October 2015 and tested for out-of-sample forecast accuracy from November 2015 to March 2016.

Many economic time series appear to be first-difference stationary, with their levels exhibiting unit root or non-stationary behavior. This can lead to a spurious regression problem, which provides statistical evidence of a linear relationship between independent non-stationary variables. Moreover, the possibility that two or more non-stationary time series might be cointegrated allows for the estimation of a long-term equilibrium relationship. Thus, the non-stationary property of the series must be considered first. Testing for cointegration is the second stage of pre-testing. The presence of a cointegrating relation forms the basis of the VECM specification. In a concept of purchasing power parity (PPP) in international trade, which defines a relationship between the nominal exchange rate and the price indices in foreign and domestic economies, a currency sometimes appears over- or undervalued, but in the absence of central bank intervention and effective exchange controls, we expect that the “law of one price” will provide some long-term anchor to these measures’ relationship.

Figure 1 below shows the historical exchange rates of EUR/DKK, EUR/GBP, EUR/PLN, and EUR/USD.

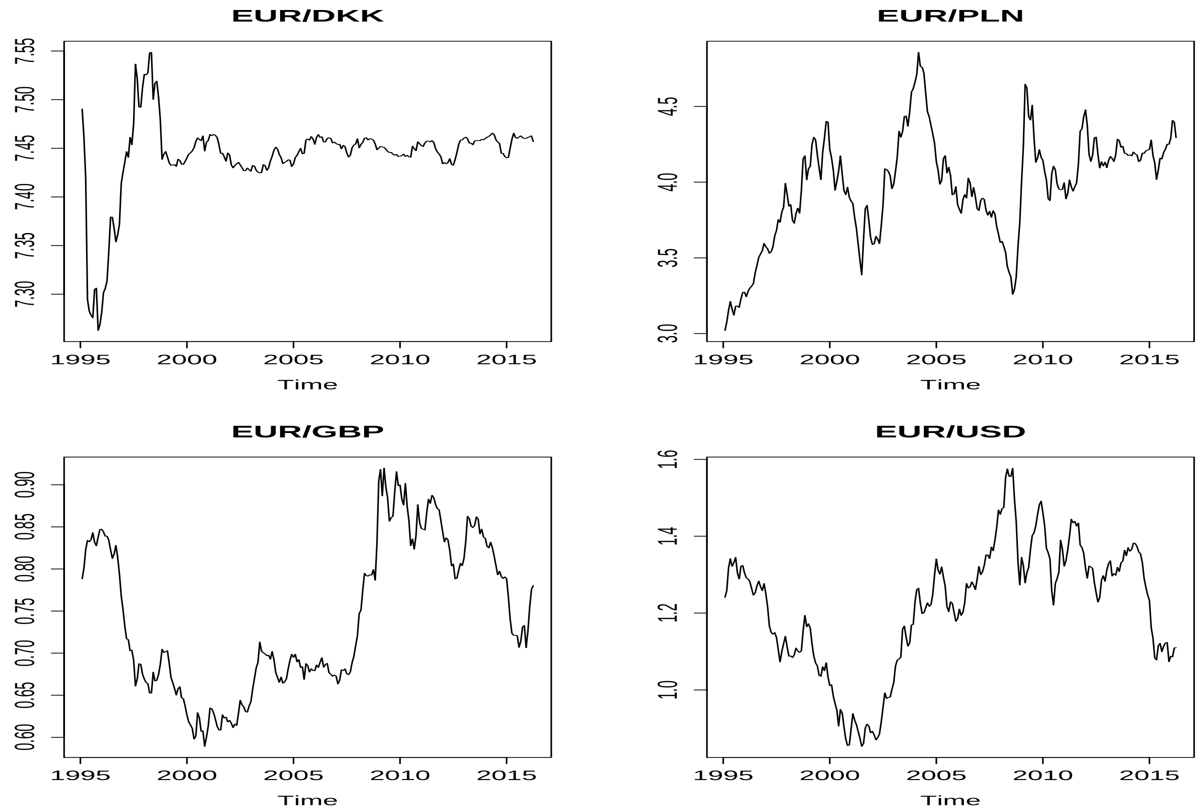


Figure 1. Historical exchange rates

According to the graphs, the exchange rate series of EUR/GBP and EUR/USD move in unison, appearing to be more volatile. So at least one cointegration relation is expected.

4. Results

4.1. Stationarity and cointegration

The stationarity test results according to the augmented Dickey-Fuller (ADF) test are reported in the Table 1. These results indicate that all series are not stationary in their level but stationary in their first difference. All series are then integrated of order one I(1).

Table 1. Results of the ADF test

Variable	Test statistic	p-value
$e_t^{EUR/DKK}$	0.1952	0.6785
$e_t^{EUR/GBP}$	-0.2120	0.5487
$e_t^{EUR/PLN}$	0.4869	0.7715
$e_t^{EUR/USD}$	-1.2703	0.2114
$\triangle e_t^{EUR/DKK}$	-10.6576	0.0100
$\triangle e_t^{EUR/GBP}$	-8.0356	0.0100
$\triangle e_t^{EUR/PLN}$	-10.2452	0.0100
$\triangle e_t^{EUR/USD}$	-10.5375	0.0100
Critical values		
10 %	-1.62	
5 %	-1.95	
1 %	-2.58	

Since all variables are I(1), we proceed to test for cointegration. We estimate a multivariate cointegration relationship to establish the existence of a long-term equilibrium relationship. The trace cointegration test was estimated with intercept in a vector autoregression (VAR) model of order 2, which was found to be the most parsimonious for the data series. Table 2 below shows the summary results of the trace statistics.

Table 2. Cointegration rank: trace statistic

Hypothesis	Test statistic	10 %	5 %	1 %
$r \leq 3$	2.66	7.52	9.24	12.97
$r \leq 2$	10.50	17.85	19.96	24.60
$r \leq 1$	24.15	32.00	34.91	41.07
r = 0	52.23	49.65	53.12	60.16

According to the table, it is evident that the trace test indicates one cointegrating equation as the null hypothesis of r = 0 is rejected at least at 10%. Thus, it may be concluded that there is a unique long-term equilibrium relationship between the variables.

4.2. Validity of the exchange rate models

1. Naive model:

   To get a stationary VAR process, the Naive model was of the form $\triangle e_t = \mu + \beta \triangle e_{t-1} + \varepsilon_t$, and Table 3 below shows the estimated results for the Naive model.

Table 3. Naive model

	Dependent variable:							
	$\triangle e_t^{EUR/DKK}$		$\triangle e_t^{EUR/GBP}$		$\triangle e_t^{EUR/PLN}$		$\triangle e_t^{EUR/USD}$	
const	4.622e-06	(0.0001)	-0.0004	(0.0010)	0.0009	(0.0013)	-0.0002	(0.0010)
$\triangle e_{t-1}^{EUR/DKK}$	2.668e-01***	(0.0620)	-0.3280	(0.6240)	0.4703	(0.7630)	-0.0660	(0.8430)
$\triangle e_{t-1}^{EUR/GBP}$	-6.934e-03	(0.0080)	0.2780***	(0.0760)	0.1518	(0.0920)	0.2369**	(0.1020)
$\triangle e_{t-1}^{EUR/PLN}$	-2.746e-03	(0.0050)	0.0460	(0.0490)	0.3568***	(0.0600)	-0.0590	(0.0670)
$\triangle e_{t-1}^{EUR/USD}$	1.510e-03	(0.0050)	-0.0770	(0.0530)	-0.0481	(0.0650)	0.2154***	(0.0710)

Note: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

From the results, each exchange rate movement is explained by its own past value, i.e. the lag of the corresponding exchange rate is statistically significant, except for the EUR/USD exchange rate equation, which has two statistically significant variables EUR/GBP and EUR/USD past values. In order to perform a residual analysis, Table 4 shows the results of the Portmanteau test for serially correlated errors.

Table 4. Pormanteau test for the Naive model

Chi-squared	p-value
214.7194	0.8783

According to the results, test statistic shows no serial correlation, i.e. the Naive model is adequate.

2. VECM:

   Table 5 below shows the estimated results for a vector error correction model.

Table 5. VECM with $r = 1$

	Dependent variable:							
	$e_t^{EUR/DKK}$		$e_t^{EUR/GBP}$		$e_t^{EUR/PLN}$		$e_t^{EUR/USD}$	
ect1	-0.060513***	(0.016)	0.36388**	(0.166)	0.67089 ***	(0.201)	-0.08280	(0.227)
$e_{t-1}^{EUR/DKK}$	0.285181***	(0.061)	-0.43712	(0.620)	0.26211	(0.750)	-0.04021	(0.846)
$e_{t-1}^{EUR/GBP}$	-0.002312	(0.008)	0.25088***	(0.076)	0.09928	(0.092)	0.24348**	(0.104)
$e_{t-1}^{EUR/PLN}$	-0.004206	(0.005)	0.05418	(0.049)	0.37573***	(0.059)	-0.06179	(0.067)
$e_{t-1}^{EUR/USD}$	-0.003697	(0.005)	-0.06407	(0.053)	-0.02421	(0.064)	0.21254***	(0.072)

Note: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

The VECM allows the long-term behavior of the endogenous variables to converge to their long-term equilibrium relationship while allowing a wide range of short-term dynamics. The coefficient of the error correction term of the EUR/DKK exchange rate variable (denoted as etc1 in the Table 5) carries the correct sign and is statistically significant at 1 percent, with the speed of convergence to the equilibrium of 6 percent. Thus, in the short-term, the EUR/DKK exchange rate is adjusted by 6 percent of the past year's deviation from the equilibrium. It confirms the stability of the system. The coefficient of the error correction term of the EUR/GBP exchange rate has a positive sign and is statistically significant at a 5 percent level. It implies that in case of any disturbance in the system, divergence from the equilibrium will take place, and the system will be unstable. The coefficient of the error correction term of the EUR/PLN exchange rate is positive and significant at a 1 percent level. As in the previous system, the EUR/PLN exchange rate will be unstable. The coefficient of the error correction term of the EUR/USD exchange rate carries a negative sign, but it is not significant. The significant coefficients of the error correction terms for each time series depict that they all cause one another in the long run. As for the short-term dynamics, every exchange rate is explained by its own past value, and other variables are statistically insignificant, except for the EUR/USD exchange rate, which is explained by the past values of the EUR/GBP and EUR/USD exchange rates. Table 6 shows the results of the Portmanteau test for serially correlated errors.

Table 6. Pormanteau test for VECM

Chi-squared	p-value
214.7675	0.726

According to the results, test statistic shows no serial correlation, i.e. the VEC model is adequate.

3. Flexible price monetary model:

   The flexible-price monetary model assumes that prices are perfectly flexible. Consequently, changes in the nominal interest rate reflect changes in the expected inflation rate. A relative increase in the domestic interest rate, compared to the foreign interest rate, implies that the domestic currency is expected to depreciate through the effect of inflation, which causes the demand for the domestic currency to fall relative to the foreign currency. In addition to flexible prices, the model also assumes UIP, continuous PPP and the existence of stable money demand functions for the domestic and foreign economies. The model further implies that an increase in the domestic money supply relative to the foreign money supply would lead to a rise in domestic prices and depreciation of the domestic currency to maintain PPP.

   So, the vector error correction model is exploited to estimate the flexible price monetary model (FPMM). Table 7 below shows estimated results for the FPMM.

Table 7. Flexible price monetary model

	Dependent variable:			
	$e_t^{EUR/DKK}$	$e_t^{EUR/GBP}$	$e_t^{EUR/PLN}$	$e_t^{EUR/USD}$
ect1	-0.2422*** (0.0267)	0.4760 (0.306)	1.0860*** (0.383)	-0.0860 (0.437)
$e_{t-1}^{EUR/DKK}$	0.1520*** (0.0560)	0.2914 (0.649)	1.2561 (0.812)	0.1157 (0.927)
$e_{t-1}^{EUR/GBP}$	-0.0093 (0.0070)	0.1777** (0.078)	0.1513 (0.098)	0.2258** (0.111)
$e_{t-1}^{EUR/PLN}$	-0.0029 (0.0040)	0.0474 (0.051)	0.3053*** (0.063)	0.0650 (0.072)
$e_{t-1}^{EUR/USD}$	0.0038 (0.0040)	-0.0804 (0.053)	-0.0718 (0.067)	0.1903** (0.076)
$m_{t-1}^{DKK-EUR}$	0.0068** (0.0030)	0.0125 (0.032)	-0.0736* (0.040)	0.0039 (0.046)
$m_{t-1}^{GBP-EUR}$	-0.0056*** (0.0020)	0.0209 (0.024)	0.0662** (0.030)	0.0198 (0.034)
$m_{t-1}^{PLN-EUR}$	-0.0004 (0.0023)	-0.0173 (0.025)	0.0188 (0.031)	-0.0030 (0.036)
$m_{t-1}^{USD-EUR}$	0.0148*** (0.0040)	-0.0510 (0.049)	-0.0170 (0.061)	-0.0073 (0.070)
$i_{t-1}^{DKK-EUR}$	-0.0016*** (0.0003)	0.0112*** (0.004)	0.0110*** (0.004)	0.0010 (0.005)
$i_{t-1}^{GBP-EUR}$	0.0008*** (0.0002)	-0.0009 (0.002)	-0.0030 (0.003)	-0.0001 (0.003)
$i_{t-1}^{PLN-EUR}$	0.0002*** (0.0001)	-0.0002 (0.001)	-0.0019** (0.001)	-0.0011 (0.001)
$i_{t-1}^{USD-EUR}$	0.0001 (0.0002)	-0.0026 (0.002)	0.0017 (0.003)	0.0001 (0.003)
$y_{t-1}^{DKK-EUR}$	0.0065 (0.0040)	0.0335 (0.048)	0.0415 (0.060)	-0.0081 (0.068)
$y_{t-1}^{GBP-EUR}$	-0.0197*** (0.0070)	-0.1470* (0.075)	0.0860 (0.093)	-0.0320 (0.107)
$y_{t-1}^{PLN-EUR}$	-0.0010 (0.0020)	-0.0018 (0.018)	-0.0020 (0.023)	-0.0122 (0.026)
$y_{t-1}^{USD-EUR}$	0.0036 (0.0070)	-0.0586 (0.082)	0.1439 (0.103)	-0.0635 (0.117)

Note: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

Remark: e_t: exchange rate, m_t: M1, i_t: short-term interest rate, y_t: industrial production index

The interpretation of the results is the same as for the VECM, but including exogenous variables, the effect of coefficients changes. The coefficient of the error correction term of the EUR/DKK exchange rate variable has a negative sign and is significant at a 1 percent level, with the speed of convergence to equilibrium of 24 percent. Compared to the VECM (see Table 5), the restoration to equilibrium path will take a shorter time because the value of the ECT is four times bigger than in the VECM model. However, the EUR/GBP and EUR/USD exchange rate coefficients of the error correction term are not significant. The coefficient of the error correction term of EUR/PLN has a positive sign and is significant, which means that the system will not be stable. As for the short-term EUR/DKK exchange rate, it is explained by the past values of differences in euro and Denmark, UK, Poland interest rates, differences in euro and Denmark, UK, USD M1 variable, difference in the euro and UK industrial production index and Denmark exchange rate, which are statistically significant at a 5 percent level. Other exchange rates, such as EUR/GBP and EUR/PLN, are explained by less fundamental variables, and the EUR/USD exchange rate is not explained by any fundamental variable. Table 8 shows the results of the Portmanteau test for serially correlated errors.

Table 8. Pormanteau test for FPMM

Chi-squared	p-value
254.9455	0.1063

According to the results, test statistic shows no serial correlation, i.e. the flexible price monetary model is adequate.

4. DCC-GARCH:

Table 9 below shows the estimated results for DCC-GARCH(1,1).

Table 9. Estimated results for DCC-GARCH(1,1)

DCC-GARCH(1,1)								
dcca	0.057927***	(0.01460)						
dccb	0.896726***	(0.02493)						
	Dependent variable:							
	$\triangle e_t^{EUR/DKK}$		$\triangle e_t^{EUR/GBP}$		$\triangle e_t^{EUR/PLN}$		$\triangle e_t^{EUR/USD}$	
μ	0.000003	(0.00005)	-0.000867	(0.00121)	0.001241	(0.00117)	0.000279	(0.00156)
α_0	0.000000	(0.00000)	0.000120***	(0.00003)	0.000024	(0.00002)	0.000098	(0.00007)
α_1	0.329880*	(0.18157)	0.348709***	(0.13429)	0.252289***	(0.08380)	0.089920*	(0.04750)
β_1	0.668070***	(0.10199)	0.234900*	(0.14280)	0.713620***	(0.08640)	0.731089***	(0.13520)
Note:								$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

What we care about the most is the joint significance of (1) α_1 and β_1 for each of the series and (2) the joint conditional correlation parameters significance of DCCA and DCCB (corresponds to a and b in $Q_t = (1-a-b)\bar{Q} + a\varepsilon_{t-1}\varepsilon_{t-1}^T + bQ_{t-1}$) because

- (1) tells whether the GARCH(1,1) “makes sense” for the given series. If α_1 and β_1 are jointly insignificant, it might be better off using constant conditional variance rather than GARCH(1,1).
- (2) tells whether DCC “makes sense” for the system of series. If DCCA and DCCB are jointly insignificant, it might be better off using a constant conditional correlation model rather than DCC(1,1).

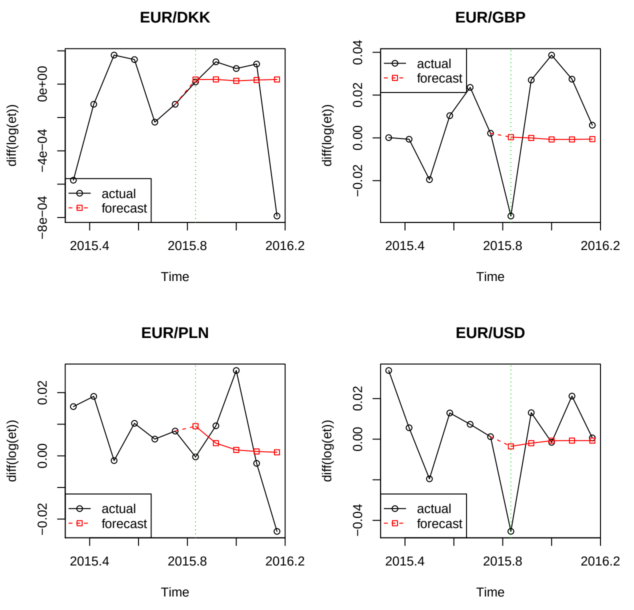
μ is the intercept of the conditional mean model ($\triangle e_t = \mu_t + a_t$) and α_0 is the intercept of the GARCH(1,1) model ($\sigma_{it,t|t-1} = \alpha_{i0} + \alpha_{i1}a_{i,t-1}^2 + \beta_{i1}\sigma_{i,t-1}$).

According to the results, all α_1 and β_1 are statistically significant for every exchange rate at least at 10 percent level. Moreover, conditional correlation parameters DCCA and DCCB are also statistically significant at a 1 percent level.

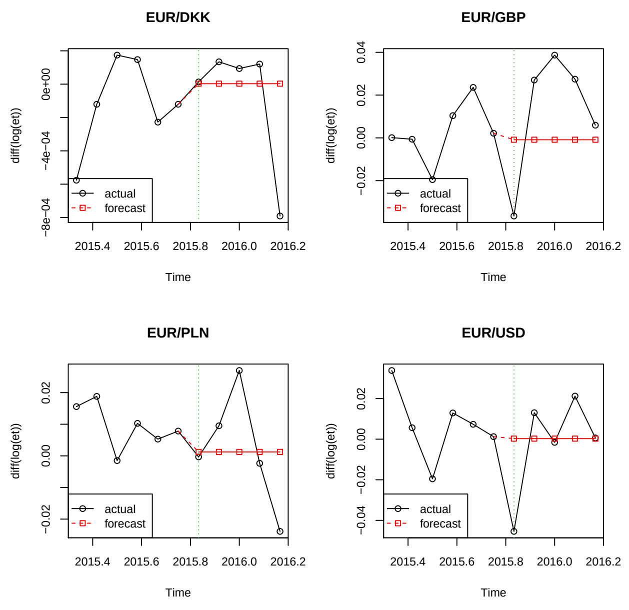
4.3. Forecast performance

- Naive model and DCC-GARCH(1,1) models:

  Figure 2 displays the actual and forecast data for Naive and DCC-GARCH(1,1) models.



(a) Naive model



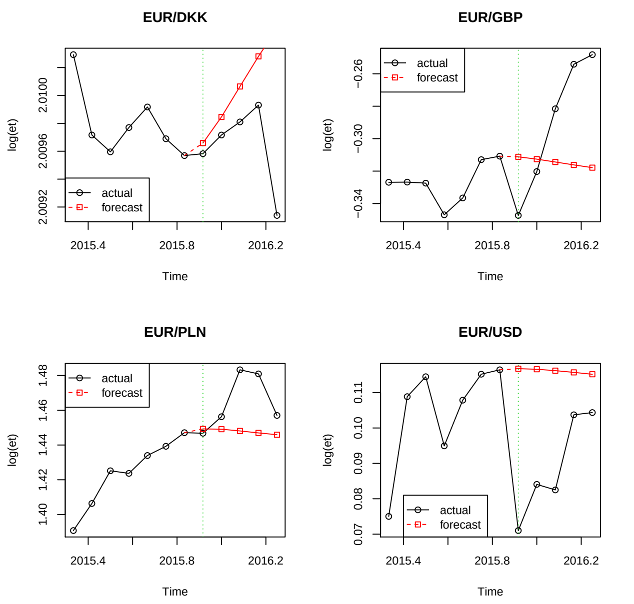
(b) DCC-GARCH(1,1)

Figure 2. Forecast performance

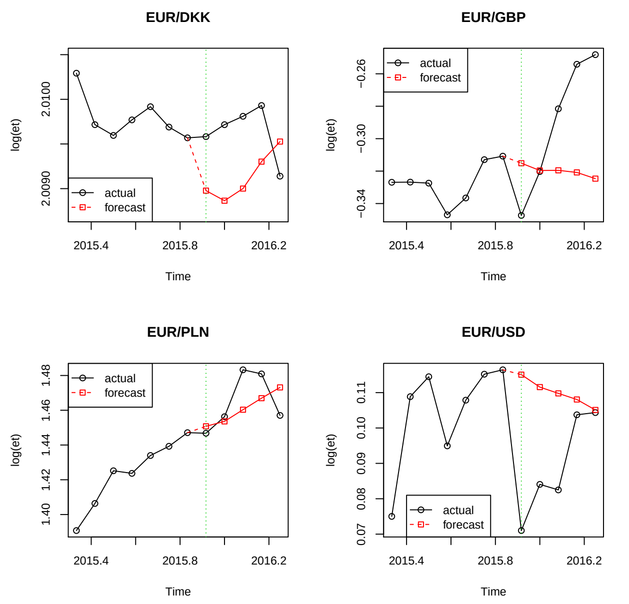
According to the graphs, the forecasting performance of these models is quite similar. Only in the EUR/PLN exchange rate case, the Naive model forecasts increase in a 1-month period, while DCC-GARCH(1,1) a decrease of the same period.

- VECM and flexible price monetary model:

  Figure 3 displays the actual and forecast data for the VEC and flexible price monetary models.



(a) VECM



(b) Flexible price monetary model

Figure 3. Forecast performance

In this respect, the forecast performance of the VEC and flexible price monetary models is variant. In the EUR/DKK exchange rate case, VECM forecasts an upward trend, while the flexible price monetary model forecasts a pit in the EUR/DKK exchange rate behavior. It shows that the inclusion of fundamental variables in the vector error correction model changes the forecast performance of the exchange rate models.

In order to compare different approaches, several accuracy measures are computed and placed in Table 10.

Table 10. Forecast accuracy measures for different models, 5-month horizon

		ME	RMSE	MAE
Naive model	EUR/DKK	-0.0001	0.0004	0.0003
	EUR/GBP	0.0126	0.0298	0.0274
	EUR/PLN	-0.0001	0.0165	0.0132
	EUR/USD	-0.0021	0.0233	0.0166
VECM	EUR/DKK	-0.0003	0.0005	0.0003
	EUR/GBP	0.0224	0.0456	0.0406
	EUR/PLN	0.0194	0.0249	0.0201
	EUR/USD	-0.0281	0.0309	0.0281
FPMm	EUR/DKK	0.0005	0.0007	0.0007
	EUR/GBP	0.0296	0.0505	0.0428
	EUR/PLN	0.0039	0.0142	0.0119
	EUR/USD	-0.0208	0.0263	0.0208
DCC-GARCH	EUR/DKK	-0.0001	0.0004	0.0002
	EUR/GBP	0.0134	0.0299	0.0276
	EUR/PLN	0.0007	0.0166	0.0129
	EUR/USD	-0.0027	0.0232	0.0163

According to the tables and RMSE, during a 5-month horizon, the Naive model outperforms the VEC and flexible price monetary models. It shows that a multivariate "random walk" is a better method for forecasting exchange rate movements for a time horizon of less than 12 months, which is consistent with Meese and Rogoff (1983) and Chinn and Meese (1995). Furthermore, according to RMSE, DCC-GARCH(1,1) also performs quite well, which means that the inclusion of volatilities as explanatory variables makes an improvement in forecasting the exchange rates.

5. Conclusions

The main objective of this paper was to forecast the multivariate exchange rate movements using several approaches such as a multivariate "random walk", vector error correction model and DCC-GARCH. A random walk is believed to have a stronger forecasting power and to be more accurate than other empirical models, especially during shorter horizons. As we were forecasting only for a period shorter than one year, the Naive model was poised to outperform the vector error correction and flexible price monetary models. The reasoning behind this was the short period of forecast, no persistent trend and a lot of studies advocating the superiority of a random walk.

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VALIUTŲ KURSŲ MODELIAVIMAS IR PROGNOZAVIMAS

Jovita Gudan

Santrauka. Šiame straipsnyje nagrinėjami euro valiutos kurso modeliai su Danijos, Lenkijos, JAV ir Jungtinės Karalystės valiuta. Šio straipsnio tikslas yra skirtingų modeliavimo metodų palyginimas ir modelių patikrinimas kryžminės patikros būdu. Vienas iš populiarių metodų yra kointegravimo ryšio įvertinimas, kuris yra realizuojamas per vektorinį paklaidų korekcijos modelį. Kointegravimas išreiškia ilgalaikius ryšius tarp nominalaus valiutos kurso ir pagrindinių kintamųjų. Šio straipsnio įrodymas pateikia išvadą, kad kryžminės patikros būdu atlikta analizė parodo, kad nominali valiutų kursų prognozė taikant *naivų* modelį gali būti pranašesnė už vektorinį paklaidų korekcijos modelį trumpesniam nei vienerių metų prognozavimo laikotarpiui.

Reikšminiai žodžiai: valiutos kursas, atsitiktinis klaidžiojimas, vektorinis paklaidų korekcijos modelis, prognozė.