VILNIUS UNIVERSITY

FACULTY OF MATHEMATICS AND INFORMATICS

MODELLING AND DATA ANALYSIS MASTER'S STUDY PROGRAMME

Master's thesis

Modeling Shadow Economy

Šešėlinės ekonomikos modeliavimas

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Vilnius, 2021

Contents

Abbreviations

- SEM - structural equation model;
- MIMIC - multiple indicators and multiple causes model;
- DYMIMIC - dynamic multiple indicators and multiple causes model;
- EMIMIC - error correction MIMIC;
- CDM - currency demand model;
- SHM - structured hybrid model;
- ECM - error correction model;
- MLE - maximum likelihood estimation;
- RFIML - restricted full information maximum likelihood function.

Abstract

J. Zajankauskaitė. Modeling shadow economy: master thesis/ supervisor Prof., Habil. dr. Vydas Čekanavičius; Vilnius university, faculty of Mathematics and Informatics, department of Statistical Analysis.

The scope of this master thesis includes the analysis of the shadow economy in Lithuania covering the period from 2000 Q1 to 2020 Q2. The study focuses on the problem related to an unobserved variable that cannot be measured directly. Shadow economy as immeasurable economic phenomenon might be an urgent economic problem causing serious consequences on the official economy of the country. Therefore, the aim of the master thesis is to identify the size of the shadow economy in Lithuania and to review determinants significantly influencing the shadow economy. As a result, three approaches of estimation of the extent of the shadow economy have been proposed.

Currency demand model, multiple indicators, multiple causes model and newly developed approach of hybrid CDM-MIMIC exposed that the shadow economy in Lithuania tends to shrink over the period for 2000 to 2020 (except recession time and period for 2019-2020). Also, the output of the econometric modeling has presented that the main causal factors affecting the shadow economy are tax burden and indicators reflecting general economic situation in the country (GDP, unemployment rate, short-run interest rate, inflation, wages, etc.).

The results of this paper serve to get a better understanding about the main tendencies of Lithuanian shadow market as well as help to identify principal instruments of controlling shadow economy in the country for policy makers.

Key words:[1] shadow economy; taxes; MIMIC; latent variable; currency demand model.

[1]The most frequent words were selected based on word cloud using text mining tools.

Santrauka

J. Zajankauskaitė. Šešėlinės ekonomikos modeliavimas: magistro baigiamasis darbas/ vadovas Prof., Habil. dr. Vydas Čekanavičius; Vilniaus universitetas, Matematikos ir informatikos fakultetas, Statistinės analizės katedra.

Magistro darbas apima šešėlinės ekonomikos Lietuvoje analizę tiriamuoju laikotarpiu nuo 2000 m. K1 iki 2020 m. K2. Darbe analizuojama problema, susijusi su latentiniu kintamuoju, kurio negalima tiesiogiai išmatuoti. Šešėlinė ekonomika kaip neišmatuojamas ekonominis reiškinys yra aktuali ekonominė problema, galinti sukelti rimtas pasekmes oficialiai šalies ekonomikai. Todėl magistro darbo tikslas yra nustatyti Lietuvos šešėlinės ekonomikos dydį bei įvardinti pagrindinius šį dydį lemiančius veiksnius. Šiam tikslui įgyvendinti buvo pasiūlyti trys šešėlinės ekonomikos masto vertinimo metodai.

Pinigų paklausos modelis, daugelio priežasčių ir daugelio indikatorių modelis bei naujai siūlomas hibridinis pinigų paklausos - MIMIC modelis parodė, kad analizuojamu laikotarpiu nuo 2000 m. iki 2020 m. Lietuvos šešėlinė ekonomika mažėjo (išskyrus recesijos tarpsnį ir 2019-2020 periodą). Be to, ekonometrinio modeliavimo rezultatai atskleidė, kad pagrindiniai priežastiniai veiksniai darantys įtaką šešėlinei ekonomikai yra mokesčių našta bei rodikliai atspindintys bendrą šalies ekonominę situaciją (BVP, nedarbo lygis, trumpalaikė palūkanų norma, infliacija, darbo užmokestis ir kt.).

Šio tyrimo rezultatai padeda geriau suprasti Lietuvos šešėlinės rinkos tendencijas, taip pat leidžia identifikuoti pagrindines šešėlinės ekonomikos kontroliavimo priemones šalyje.

Raktiniai žodžiai:[2] šešėlinė ekonomika; mokesčiai; MIMIC; latentinis kintamasis; pinigų paklausos modelis.

[2]Raktiniai žodžiai pasirinkti pagal žodžių debesį, naudojant teksto gavybos įrankius.

Introduction

Discussions of the shadow economy phenomenon and its consequences to the official economy are widely spread in the whole world. Although the number of problems related to the shadow economy (evasion of taxes, social security fraud, budget deficit, illegal work, instability in labour market, distorted official indicators, lower principles of morale, etc.) have been extensively investigated for a long time by many authors [14, 17, 43, 45, 50], there is no clear agreement whether this complex phenomenon has more negative than positive effects on the observed economy.

As highlighted [47], the shadow economy is a natural component of social and economic life existing in all countries around the world. Actually, having 0 % of shadow economy is not possible and not even acceptable (from the point of view of economic policy, a relevant example could be zero unemployment rate in labour market) because by battling shadow economy business and entrepreneurship are attacked too. According to [28], shadow economy is a certain help during a recession time that may play a significant role as a "backup plan". In such a difficult period, shadow economy can be treated as a powerful tool for boosting the overall production of goods and services since income earned in the shadow economy is spent in the official economy. Indeed, as shown in [1], a positive relationship between growth of the GDP and growth of the shadow economy exists. However, it is worth to notice that unobserved economy might be very dangerous and damaging in a long-run due to one simple reason: shadow economy tends to create more shadow economy. Hence, shadow economy varies among two extremeness:

1. it is the reason for social, microeconomic or even macroeconomic problems of the country;
2. it is a free space for entrepreneurship fighting with inequality in the tax system and strong state regulatory policy.

Although it is difficult to evaluate shadow economy's impact on the formal economy, public authorities have to take into account the size of the informal economy when providing statistical information about the overall economic situation in the country or especially setting goals of monetary and fiscal policies. Thus, this thesis is aimed to identify the extent of the shadow economy in Lithuania over the period for 2000 to 2020 and to highlight the main factors having influence on it.

The purpose of master thesis is achieved by accomplishing the main tasks of the work:

1. To define the main characteristics of the shadow economy and to reveal the concept of this complex phenomenon.
2. To identify causal factors and indicators of the shadow economy, herewith to evaluate the development of the shadow economy over analyzed period.
3. To fit CDM and MIMIC models on gathered data of Lithuania, to apply a new approach of hybrid CDM - MIMIC, to disclose drawbacks and strengths of each method.

4. To compare the results of the different estimation methods.

This master thesis is structured as follows. First section introduces the review of the literature. The main focus of this section is on theoretical background for this study covering the definition of the shadow economy, forces that lead to the unobserved economy, estimation methods. Second section produces methodology of empirical research. Third part presents data description and shows estimation results, lays out the causes of the shadow economy and its indicators, provides the size of the shadow economy as the share of shadow economy in total GDP. Finally, the most important findings and the main conclusions are presented.

1. Literature review

1.1. Conceptual framework of the shadow economy

As it was mentioned earlier, shadow economy could become a hard challenge for both economic and social policy. For that reason, this problem is extensively explored by policymakers, economists and econometricians. In [46], it is argued that estimation of the shadow economy development could be appointed as scientific passion for knowing the unknown. Many comprehensive articles analyzing the topic of master thesis could be found, nevertheless, there is no agreement of what estimation method or definition could be treated as most adequate. Therefore, more detailed investigation on these controversial questions is needed.

1.1.1. Definition of the shadow economy

Results of empirical investigation may strongly depend on strict rules how to determine shadow economy. However, several difficulties are faced when shadow economy is characterized. Firstly, there is no commonly known scientific definition established and secondly, the conception might vary on a chosen field and measurement methods of the research.

At the first point it may be fair to ask why the particular word "shadow" is used to define analyzed occurrence in the economy. Shadow is derived from a physical concept and perceived as a dark area to which light rays do not enter due to an obstacle in their path. The origins of the shadow economy as an important economic phenomenon could be detected back in the 17th century. A well-known economist and the founder of the classical school of economics A. Smith has stated that economic agents act in their self-interest. Nevertheless, in the right environment, these selfish aspirations can yield benefits for society [48]. A more detailed concept of the informal economy has begun to develop in the 20th century when the shadow economy was described by V. Tanzi as actions of individuals and firms to involve in economic activities without the intervention of the government [51]. Slightly later, A. Smith formulated that shadow market generates legal or illegal provision of good and services that is not included in GDP [49].

However, many authors in their works [40, 46, 57, 58] agree that illegal activities of a black market such as drug dealing, smuggling, gambling, fraud, etc. should be excluded from definition. The main motive of eliminating these unauthorized actions from terminology is that quite often a too "big" size of the shadow is measured. As a result, a more narrow definition will continue to be used in further analysis of this master thesis.

Thus, despite the fact that global definition of shadow has not been found yet, it could be concluded that the shadow economy reflects all legal production of goods and services that are hidden from official authorities and that are not captured in national statistics.

1.1.2. Drivers determining the extent of the shadow economy

In the 1980s, countries of the European Union were facing dramatically rising unemployment, budget deficit, disappointment about social and economic strategies, herewith vagueness and anxiety in the labour market. Thereby, attention was being drawn to the informal economic since it was blamed for many problems listed above. Furthermore, in recent years, a plenty of conducted studies have indicated that the peculiarities of shadow economy's leading factors is not a taboo topic anymore, on the contrary, it is an essential part of contemporary economies.

Factors determining the size of the shadow economy may be grouped into causes and indicators (consequences). Causes indicate the strength of their impact on the shadow economy while a change in the extent of the shadow economy is reflected by the indicators. Although the causes and the consequences differ in each country, general systemized information from different articles is going to be introduced. Also, from the economic point of view, interpretation and identification of adequate signs of factors will be presented. Hence, firstly, a description of the general causes leading to the phenomenon is examined, then indicators are analyzed.

Potential causes and their effects on informal economy:

- **Tax rates and social security contribution burden.** It is a key determinant and one of the most commonly used covariate in the analysis. A decreasing burden of taxation has a positive impact on the decision of economic agents to operate in official economy. Accordingly, the size of the shadow economy reduces when tax rates fall. Nevertheless, in [30] it was shown that lower taxes were associated with higher shadow activity because of inferior quality of the public services. In addition, as it was stated in [56], the higher the tax morale, the less likely is participation in the shadow economy.

- **Unemployment rate.** The fluctuations of unemployment level in the labour market could have both positive and negative effects on the market of shadow economy. As shown by [12], unemployment rate has a negative and statistically significant impact on the size of the shadow economy in the short-run, while the unemployment rates has a positive impact in the long-run. Also, other authors in their works [5, 17, 53, 55] agree that identification of correlation between shadow activities and unemployment rate is ambiguous. However, based on economic logic, this labour market indicator is considered as one of the best incentives for an individual to engage in the shadow activities in order to improve financial status.

- **Interest rate.** If the interest rate increases, the cost of borrowing and incentive to save rather than spend increase, therefore economic agents try to earn by holding money in banks or other financial institutions. As a result, the amount of cash in circulation decreases, which corresponds to the decline of the shadow economy. Indicator of interest rate reflects the opportunity cost of holding cash. Consequently, negative coefficient is expected.

- **Average earnings.** As in the case of the unemployment rate, it is quite difficult to implicate the sign of this relationship. As [29, 33] pointed out, the increase in the minimum wage can increase the likelihood of becoming an informal worker, thus, the higher level of shadow economy should be observed. Also, looking from the perspective of the firms, higher wages may force to the avoidance of paying taxes and pursuit of alternative methods of circumventing authorities due to increased cost. On the other hand, when salary grows, living standards and disposable income increase. In this case, individuals have no need to operate in shadow activities and herewith to risk. To sum up, the growth of earning may be considered as a long term leading factor for the shadow economy.

- **Government budget balance.** Increasing public fiscal balance may implicate the higher government revenues from taxes. As mentioned previously, it is assumed that more taxes tend to create more shadow in the economy. Thus, positive effect is expected. More details could be found in the [7].

- **Residential property prices.** As highlighted in [24], real estate market is also affected by shadow economy since informal activities tend to push up house prices. It is assumed that positive correlation between these two indicators should exists.

- **Inflation.** The widely used measurement of inflation is harmonised index of consumer prices. In the context of macroeconomic theory, inflation may be treated as a procyclical indicator [36] since it tends to decrease during an economic downturn and to increase during an economic booms. The rise in the level of prices in an economy causes a reduction in the purchasing power. In this case, the growth of the economic agents' shadow economic activities can be expected due to the lost utility in the formal economy. Thus, a positive relation between inflation and the shadow economy might be expected. The evidence of that positive correlation is provided in [37]. On the other hand, if the shadow economy is linked with cash money in the official economy, a reduction of purchasing power and herewith the lower amount of the cash in the money market could be recorded (the lower the demand for cash payments and the higher demand for credit payments, the lower the shadow economy). The impact of inflation on the underground economy in opposite direction is explained in [2] where it is said that inflation leads economic agents to operate more with credit. Moreover, rising inflation encourages a smaller volumes of currency holdings.

Also, a broad range of other factors that could drive the shadow economy (regulatory burden, corruption, trade openness, disposable income, firm's profitability, etc.) may be discovered in the literature. Furthermore, another important category of factors capturing and reflecting changes in shadow economy must be clarified.

Potential indicators:

- **GDP**. State of the official economy could be considered as one of the major indicator. The effects of the shadow economy on official economy and vice versa vary greatly depending on the case of the analyzed country. For instance, in Greece it was found that GDP strongly correlates (a positive correlation exists) with shadow economic activities [6]. On the contrary, rising economy reduces the extent of shadow economy in Ukraine [23]. Economically speaking, when negative relationship is considering among GDP and shadow economy, it is assumed that countries experiencing a shrinkage in official GDP are able to reduce and cover a drop through growth of the informal economy. To summarize, there are different approaches and interpretations of the relationship between formal and informal economics.
- **Monetary indicators**. It is known that mostly shadow activities are paid in cash. To take this into account, the relationship among money aggregates (M0[3], M1[4], M2[5], M3[6]) and shadow economy should be positive.

Additionally, other authors in their works evaluated shadow economy from working hours [46], labour force participation rate or growth rate of the total labour force [44], etc.

Theoretically derived causes and consequences of shadow economic activities have to be investigated empirically. In the next subsection, methods to estimate the size of the shadow economy is going to be revealed.

[3] Monetary base.

[4] Currency in circulation and overnight deposits in euro and foreign currencies.

[5] M1 and other short-term deposits in euro and foreign currencies.

[6] M2 and marketable instruments.

1.2. Classification of the estimation methods

In general, according to the Professor F.Schneider, three estimation methods may be specified [46]:

1. Direct methods.
2. Indirect methods.
3. Causal or latent estimation methods (statistical models).

Each of the method is disclosed in the next chapter of the master thesis.

1.2.1. Direct approach

The main characteristics of direct approaches are identified as follows: firstly, it is a procedures based on the microeconomic level data and secondly, results by this method could be obtained only at one particular point in time (point estimates).

Such methods as interviews, sample surveys, tax auditing, expert evaluations, discrepancies in National Accounts, etc. empower detailed information to be gathered, provide a structure and main parts of the shadow economy. Regardless, the methods of direct approach suffer from some drawbacks. Estimates of the shadow market might be biased due to the lack of sample representativity or extremely sensitive questions in the questionnaire. Quite often respondents engaged in these activities do not wish to be identified or do not want to tell the truth. Hence, the risk of non-response problem is arised. As a result, shadow economy is likely to be both underestimated or overestimated.

The application of the direct method to assess the size of the shadow economy in Lithuania is presented in [40]. Authors carry out research standing on surveys of entrepreneurs, investigate causes and effects of "envelope" wages, unreported business income, illegal workers, etc. It was found that the lowest tolerance of tax evasion and tax morale is in Lithuania (comparing to Latvia and Estonia). Also, the level of bribery is less tolerated in neighbour countries.

1.2.2. Indirect approach

The use of macroeconomic indicators (national income and expenditure, GDP, wages, unemployment, etc.) and reflection of the magnitude of the shadow economy over time are the main features of indirect approaches. The key benefit of these methods is an ability to assess the tendency and dynamics of informal economy in a specific range of time. Nonetheless, indirect approaches are widely criticized since they are quite sensitive to the particular assumptions. Currently three classes of indirect methods have been proposed by [46]:

- **Approach via national accounting.**

  There are two major methods for a category of the approach via national accounting:

  1. The discrepancy between national expenditure and income statistics. It is assumed that gross national product should be the same as national expenditure, thus, the space between the national income and expenditure could be treated as shadow economy indicator. However, results may be unreliable due to potentially included mistakes in national accounts statistics.
  2. The discrepancy between the official and actual labour force. Labour force participation rate is stated as important indicator of the shadow economy. The main assumption is that labour force participation is considered to be constant, hence, increasing indicator implicates higher level of the shadow economy, *ceteris paribus*. Unfortunately, the discrepancy between the official and actual labour force might also be affected by other drivers.

- **Monetary methods.**

  Monetary methods are based on the assumption that activities of the shadow economy are paid in cash. In the literature there are two main approaches:

  1. The transactions approach. This approach was first used by E. Feige [19]. It is assumed constant relationship among transactions and official gross national product and no shadow economy in the base year. Also, well known Fisher's quantity equation is analyzed. This approach, too, has some disadvantages related to the strict and doubtful assumptions.
  2. The currency demand approach. The approach was first found by P. Cagan in his work [9] and developed by V. Tanzi [52]. V. Tanzi econometrically estimated a currency demand model with this assumptions:
     (a) shadow economy transactions are made in a form of cash payments (economic agents' desire not to be traced by official authorities);
     (b) non-existent shadow economy in a base year;
     (c) equality of a velocity of the money in the official and hidden economy;
     (d) tax burden is interpreted as the main reason to engage in illegal activities.

     The main idea of currency demand approach is to evaluate a model in which indicator of taxes would have a positive effect on the use of money and to identify the "excess" demand for currency. It has been noticed that the CDM is commonly used in the literature.

- **Physical input method.** Electricity consumption approach. This method is based on only one physical indicator related to the household consumption of electricity. The increase of the total electricity consumption is associated with an increase in overall GDP of formal and informal economy, where an electricity to GDP elasticity is close to one. More details and critique could be found in one of the first articles [32], [34] about the electricity consumption method.

1.2.3. Latent estimation approach

This group of estimation approaches is considered to be the most comprehensive and novel. As was mentioned earlier, statisticians assign shadow economy to the latent or unobserved category of variables. Since informal economy cannot be measured directly, statistical models (MIMIC, DYMIMIC, EMIMIC, SEM, etc.) are used in order to estimate the size of the shadow economy. These models have become very popular, often used and well appreciated by scientists since they may operate with a large number of variables at the same time. Hence, an information of international trade, labour market, money market or goods and services market could be used in modeling. For instance, if we take an example of the frequently used MIMIC model, we select both a group of causal variables and indicators simultaneously. MIMIC is a special case of SEM where the causal relationships between an unobservable and observable variables are specified by using their covariance information. Many authors use the MIMIC model to estimate the size of the shadow economy for various countries [8, 14, 55].

Another alternative of MIMIC is DYMIMIC defined as MIMIC model in first differences. Unfortunately, dynamic version of MIMIC does not protect against the loss of data's long-run information if drivers are used in their first differences. Additionally, if the variables are cointegrated and a stationary long-run relationship exists among them, EMIMIC model might be used.

To conclude, the identification of an appropriate measures of the informal economy depends on various aspects: first of all, the aim of the research; reliability of assumptions; specific features of country (e.g. general economic development, dramatic structural changes, etc.); limitations of data availability. Given these considerations, CDM, MIMIC and SHM (new approach combining CDM and MIMIC) were chosen as the most acceptable methods to be used in further analysis of Lithuania's shadow economy market.

To sum up, based on literature review, definition, leading factors and classification of estimation methods of the shadow economy were presented. Hence, there is no clear and universal explanation of how shadow economy should be defined. Most authors can admit on variety of the characteristics of what is known as the informal economy. Nevertheless, it should be noted that definition excludes all illegal activities of criminal sector. Potential causes of the shadow economy are considered as tax rates and social security contribution burden, unemployment rate, interest rate, average earnings, government budget balance, residential property prices while GDP and monetary indicators are recognized as potential indicators. Although scientific articles provide many measurement techniques for estimation of the size of the shadow economy, the effect of certain factors is still hard to be measured. Furthermore, all methods have their pros and cons.

2. Empirical methodology

In this section, econometric tools that is going to be used is introduced. The whole empirical research might be divided into three main parts which allow to create a coherent research plan (see Figure 1):

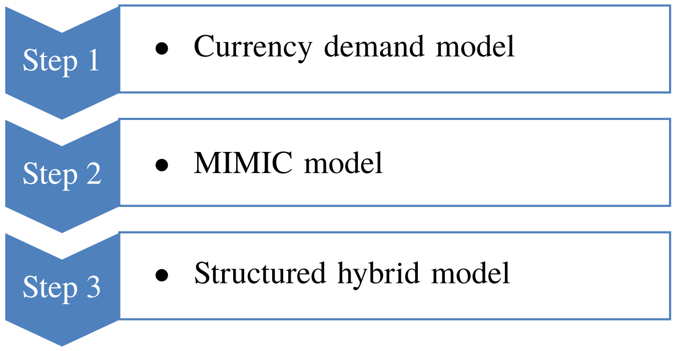

Figure 1. The plan of an empirical research.

Each structural part of the research and the methodology of the modeling process are discussed in detail in the following subsections.

2.1. Currency demand model and an error correction approach

2.1.1. Currency demand model

As previously stated, CDM is widely used for modeling shadow economy. To start with, it has to be noted that an identification of model's assumptions plays an important role in making statistical inferences. For that reason, first of all, assumptions of CDM have to be laid out. As far back as the 18th century, B. Franklin [20] has stated that in our world nothing could be said to be certain, but death and taxes. Indeed, this expression perfectly reflects one of the assumption of currency demand model that the main cause of shadow economic activities is tax burden. Moreover, in 2000, at a conference in Washington, one of the most prominent American economists B. Friedman has highlighted the importance of a cash money on shadow economy as "...cash leaves no tracks, and makes no demands on anybody else's integrity." (The Economist, July 22, 2000, p.76). This citation satisfies the main idea of another model's assumption about transactions that are only paid by cash in the shadow economy. Remaining assumptions of CDM have already presented in Subsubsection 1.2.2. Currency demand model that was first introduced by V. Tanzi [51] is the following (1):

$$\ln\left(\frac{C}{M_2}\right)_t = \alpha_0 + \alpha_1 \ln(1+T)_t + \alpha_2 \ln\left(\frac{W}{Y}\right)_t + \alpha_3 \ln R_t + \alpha_4 \ln\left(\frac{Y}{N}\right)_t + u_t \tag{1}$$

where $\frac{C}{M_2}$ is a ratio of cash to M2 money supply, T is an indicator of taxes, $\frac{W}{Y}$ is a share of wages in national income, R is an interest rate, $\frac{Y}{N}$ is GNP per capita.

The most important point of CDM lies in the excess demand for money that is mainly attributed to the rise in taxes and herewith, the growth of the shadow economy.

Since contribution of V. Tanzi [51], different versions of the CDM have been used for the estimation. Nevertheless, the core principles of steps for further modeling are going to be used based on [4].

Firstly, it is needed to estimate the following regression (the total cash demand) (2). Also, based on the assumption, positive effect of the tax burden on the amount of money must be obtained $\implies \alpha_1 > 0$.

$$\ln\left(\frac{C}{M_2}\right)_t = \alpha_0 + \alpha_1 \ln(1+T)_t + \alpha_2^\top X_t + u_t \tag{2}$$

where $\alpha_0, \alpha_1, \alpha_2$ are parameter vectors, X_t is a logarithm of a vector of all covariates except taxes.

Then, parameters of currency demand model in which the tax indicator is set equal to zero are estimated from:

$$\ln\left(\frac{C}{M_2}\right)_t^* = \alpha_0 + \alpha_2^\top X_t + u_t \tag{3}$$

Notably, a unit was added to the tax variable in Equation (2) since $T = 0 \implies \ln 1 = 0$, otherwise $\ln 0 = -\infty$. Next, difference between estimated Equation (2) and Equation (3) is treated as the size of illegal cash in the economy:

$$IM_t := \exp\left\{\hat{\alpha}_0 + \hat{\alpha}_1 \ln(1+T)_t + \hat{\alpha}_2^\top X_t + \ln(M_2)_t\right\} - \exp\left\{\hat{\alpha}_0 + \hat{\alpha}_2^\top X_t + \ln(M_2)_t\right\}$$

In order to calculate legal money in the economy, the series of illegal money is subtracted from the series of monetary aggregate M2 for each period:

$$LM_t = (M_2)_t - IM_t$$

Then, the speed rotation of cash is derived through GDP and legal money supply:

$$SR_t = \frac{GDP_t}{LM_t}$$

Following assumption about the equality of the velocity of legal and illegal money, the final estimations for the shadow economy is obtained by multiplying the illegal money with the speed rotation of cash for each respective period:

$$SE_t = IM_t \cdot SR_t$$

Finally, the share of the shadow economy in total GDP is received (shadow economy as percentage of GDP):

$$SE_t(\%) = \frac{SE_t}{GDP_t} \cdot 100\%$$

2.1.2. ECM

Further, based on other authors' articles, an error correction framework has been found to be useful in modeling the currency demand [3, 4, 39]. Since time series are analyzed, there is a likelihood of a unit root and cointegration. If the variables are non-stationary, estimates of ordinary least squares might lead to a spurious regression, hence, the standard t and F tests become deceptive. Well know Granger's representation theorem [27] states that if non-stationary variables are cointegrated then they can be characterized by an error correction mechanism. As a result, two-steps ECM procedure was suggested by Engle and Granger [18]. However, in the case of CDM, one-step ECM is considered to be more attractive (especially from the economic view) due to joint short-run and long-run estimation and identification of an long-term equilibrium between indicators (cointegration's concept reveals the equilibrium concept from economics to econometrics). Besides, one-step ECM has a certain advantage in calculating the size of the shadow economy since clearly separation among dependent and independent variables is found. The main procedures in one-step ECM:

- testing for a unit root and non-stationarity;
- taking the first differences and testing the new series for a unit root and non-stationarity again;
- if the new series is stationary then it is concluded that series is integrated of order 1;
- testing stationarity of residuals of the regression equation for the long-run currency demand and drawing conclusions about cointegration, herewith a long-run relationship between variables;
- including one period lag for dependent and independent variables;
- estimating the long-run and short-run together, checking residuals;
- identifying long-run effects (lagged one period dependent and independent variables) and short-run effects (first differenced independent variables).

Mathematically, if it is known that two variables Y_t and X_t are $I(1)$, then linear combination of Y_t and X_t exists: $\hat{u}_t = Y_t - \hat{\alpha}_1 - \hat{\alpha}_2 X_t$ where $\hat{u}_t \sim I(0)$. Thus, Y_t and X_t are cointegrated. Then, the basic approach of the cointegration and ECM could be expressed as:

$$\Delta Y_t = \alpha + \beta \Delta X_t - \gamma \hat{u}_{t-1} + e_t$$

where β reflects the short-run effect and γ is the adjustment effect (indicates how disequilibrium should be corrected).

2.2. Multiple indicators - multiple causes model

2.2.1. Theoretical principles

The roots of MIMIC model are found in the psychometric literature of the factor analysis. The first economic implementation may be detected in articles by A. Zellner [61] and A. Goldberger [25]. MIMIC was first applied to the estimation of the shadow economy in [21] and since then became very popular among economists and econometricians (the key benefit is that MIMIC considers both multiple causes and indicators of the the shadow economy at the same time). The principal idea of the MIMIC is to investigate the connections among a latent variable (the size of shadow economy) and a group of observable variables (causes and indicators) by using information in covariance matrix.

Formally, the model consists of two main parts [43]: the structural equation model (specifies causal relationships between the unobserved variables) and the measurement model (links the unobserved variable to observed indicators). The general structure of MIMIC model may be seen in Figure 2.

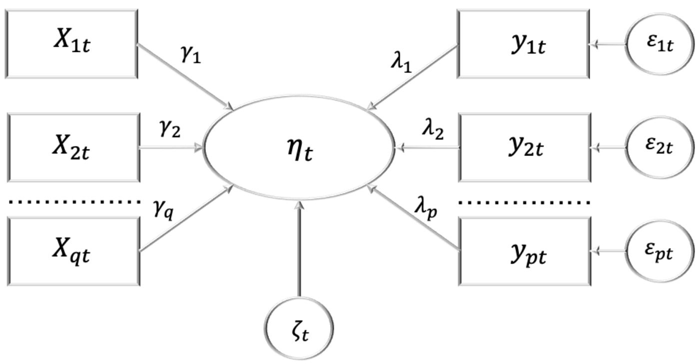

Figure 2. The structure of the MIMIC model.

The structural equation model:

$$\eta_t = \gamma^\top X_t + \zeta_t \tag{4}$$

where $X_t^\top = (X_{1t}, \dots, X_{qt})$ is a $(1 \times q)$ vector of causes;

$\gamma_t^\top = (\gamma_{1t}, \ldots, \gamma_{qt})$ is a $(1 \times q)$ vector of coefficients;

ζ_t is an error term.

Hence, unobservable η_t (the shadow economy) is a scalar variable that is linearly described by a range of observable causes and an error term.

The measurement model:

$$y_t = \lambda \eta_t + \varepsilon_t \tag{5}$$

where $y_t^\top = (y_{1t}, \ldots, y_{pt})$ is a $(1 \times p)$ vector of each endogenous indicators $y_{jt}, j = 1, \ldots, p$;

λ_j, $j = 1, \ldots, p$ is a $(p \times 1)$ vector of regression coefficients;

$\varepsilon_t = (\varepsilon_{1t}, \ldots, \varepsilon_{pt})$ is a $(p \times 1)$ vector, where every $\varepsilon_{jt}, j = 1, \ldots, p$ is a white noise term and covariance matrix $(p \times p)$ is denoted as Θ_ε.

Thereby, η_t determines a set of indicators (endogenous variables), subject to a vector of random error terms.

Also, it is assumed that in (4) and (5) ζ_t (structural noise) and the elements of ε_t (measurement error) have a normal distribution and are linearly independent.

Model assumptions:

- $E(\eta_t) = E(X_t) = E(\zeta_t) \implies$ the variables are measured as deviations from their means
- $E\left(X_t \zeta_t^\top\right) = E\left(\zeta_t X_t^\top\right) = 0 \implies$ the error terms in the structural model do not correlate to the causes
- $E(y_t) = E(\varepsilon_t) = 0 \implies$ the indicators are directly measurable and expressed as deviations from their means
- $E\left(X_t \varepsilon_t^\top\right) = E\left(\varepsilon_t X_t^\top\right) = 0 \implies$ the error terms in the measurement model do not correlate to the causes
- $E\left(\eta_t \varepsilon_t^\top\right) = E\left(\varepsilon_t \eta_t^\top\right) = 0 \implies$ the error terms in the measurement model do not correlate to the the latent variable
- $E\left(\zeta_t \varepsilon_t^\top\right) = E\left(\varepsilon_t \zeta_t^\top\right) = 0 \implies$ the error terms in the structural model do not correlate to the error terms in the measurement model
- $E\zeta_t^2 = \sigma^2 \implies$ the constant variance of the error terms in the structural equation model

MIMIC model's covariance matrix (6) is derived from (4) and (5):

$$\Sigma = \begin{pmatrix} \lambda\left(\gamma^\top \Phi \gamma + \psi\right) + \Theta_\varepsilon & \lambda \gamma^\top \Phi \\ \Phi \gamma \lambda^\top & \Phi \end{pmatrix} \tag{6}$$

where Θ_ε is $(p \times p)$ measurement error's covariance matrix;

Φ is $(q \times q)$ covariance matrix of the causes;

$\psi = Var(\zeta_t)$.

Thus, information in (6) is used for estimation of a MIMIC model where the distance between an observed covariance matrix and the covariance matrix predicted by the model is minimized. Since parameters estimation is fundamental task in modeling (the more accurate estimates the more adequate conclusions), it is worth to discuss estimation method of the MIMIC model.

In a nutshell, the main idea of an estimation of a MIMIC is that covariance information of sample data is used to get estimates of population parameters. Therefore, the population covariance matrix is:

$$\Sigma = \Sigma(\theta)$$

where θ is a vector of the parameters of the model ($\theta = f(\lambda, \gamma, \psi, \Phi, \theta)$).

As a result, estimation is started by obtaining an estimate of the population covariance matrix $\hat{\Sigma}$ that should be as close as possible to the sample covariance matrix S. Based on [42], maximum likelihood estimation (MLE) is going to be applied for MIMIC model and the following fitting function is going to be minimized:

$$\mathrm{F_{ML}} = \log|\Sigma(\theta)| + \mathrm{tr}\left[S\Sigma^{-1}(\theta)\right] - \log|S| - (p+q)$$

where S is the sample covariance matrix having Wishart distribution; log| | is the log of the determinant of the respective matrix; $(p+q)$ is the number of observed variables; S and $\Sigma(\theta)$ are positive definite, nonsingular.

Finally, estimators $\hat{\lambda}, \hat{\gamma}, \hat{\psi}, \hat{\Phi}, \hat{\theta}$ are obtained by applying iterative numerical procedures.

2.2.2. Goodness-of-fit statistics

The performance of model fit indices helps to assess how successfully model fits the sample data. The adequacy of MIMIC model is tested with a number of criteria. The formulas that are going to be used in the empirical part are presented in this subsubsection based on [10]. Hence, the most commonly used goodness-of-fit statistics are:

1. χ^2 - Chi-square test statistic:

$$\chi^2 = -2\left\{-\frac{1}{2}(n-1)\left[\mathrm{tr}\left(\mathrm{S}\Sigma^{-1}\right) + \log|\Sigma| - \log|\mathrm{S}| - p\right]\right\} = (n-1)\mathrm{F}$$

where p is the number of observed variables; S is a sample covariance matrix.

The smaller the likelihood related to χ^2, the worse fit between the perfect model and the hypothesized model. The χ^2 value and model degrees of freedom can be used to calculate a p-value. If p-value is less than 0.05 then the model deviation from the data is significant at the 5 % significance level $\implies$ the model is not properly specified and predictions do not match the actual data.

2. $SRMR$ - Standardized Root Mean Square Residual index:

$$SRMR = \sqrt{\frac{\sum_{i=1}^{p}\sum_{j=1}^{i}\left[\left(s_{ij} - \hat{\sigma}_{ij}\right) / \left(s_{ii}s_{jj}\right)\right]^2}{p(p+1)/2}}$$

where p is the number of observed variables; s_{ij} is a component of S; $\hat{\sigma}_{ij}$ is a component of $\Sigma(\hat{\theta})$.

$0.05 < SRMR < 0.1 \implies$ acceptable fit. $SRMR \leq 0.05 \implies$ good fit.

3. $RMSEA$ - Root Mean Square Error of Approximation index:

$$RMSEA = \sqrt{\max\left\{\left(\frac{\mathrm{F}(\mathrm{S}, \Sigma(\hat{\theta}))}{\nu} - \frac{1}{n-1}\right), 0\right\}}$$

where $\mathrm{F}(\mathrm{S}, \Sigma(\hat{\theta}))$ is the fit function; $\nu = l - t$ is the value of degrees of freedom (l is a number of known parameters, t is a number of independent parameters); n is a sample size.

$0.05 < RMSEA < 0.08 \implies$ a fit close to good. $RMSEA \leq 0.05 \implies$ good fit.

4. TLI - Tucker-Lewis Index:

$$TLI = \frac{(\chi_i^2/\nu_i) - (\chi_t^2/\nu_t)}{(\chi_i^2/\nu_i) - 1} = \frac{(\mathrm{F}_i/\nu_i) - (\mathrm{F}_t/\nu_t)}{(\mathrm{F}_i/\nu_i) - (1/(n-1))}$$

where χ_i^2 is connected with the independence model and χ_t^2 is connected with the target model; ν_i is degrees of freedom for the independence model and ν_t is degrees of freedom for the target model; n is a sample size.

$0.95 < TLI < 0.97 \implies$ a fit is more acceptable compared to the independence model. $TLI \geq 0.97 \implies$ fit is strongly preferable compared to the independence model.

5. CFI - Comparative Fit Index:

$$CFI = 1 - \frac{\max\left[\left(\chi_t^2 - \nu_t\right), 0\right]}{\max\left[\left(\chi_t^2 - \nu_t\right), \left(\chi_i^2 - \nu_i\right), 0\right]}$$

where χ_i^2 is connected with the independence model and χ_t^2 is connected with the target model; ν_i is degrees of freedom for the independence model and ν_t is degrees of freedom for the target model.

$0.95 < CFI < 0.97 \implies$ acceptable fit. $CFI \geq 0.97 \implies$ good fit.

The general consensus is that a smaller SRMR and RMSEA, larger CFI and TLI and smaller than 0.05 p-value implicate better fit. In this case, there is no necessity to modify the models unless the signs of coefficients are not appropriate to the economic interpretation.

2.2.3. Criticism of the MIMIC

The key points of criticism covering the details about definition and data, instability of coefficients and benchmarking are presented below.

Definition and data. As was mentioned earlier, shadow economy definition does not include illegal activities that fit attributes of classical crimes. However, MIMIC-based shadow economy estimates might be suspected to cover these illegal actions. Thus, it is difficult to ensure no data duplication and not too high macroeconomic value of the shadow economy. Also, the risk of inappropriate choice on causal and indicator variables exists. As a consequence, a completely different latent variable than the one of interest can be estimated.

Instability of coefficients. Estimation is highly sensitive with the respect to changes in the data. The stability of the coefficients is lower when having a small sample size. In this case, the sample covariance matrix does not converge to the theoretical covariance matrix of the model. Nevertheless, as was shown by [13], instability vanishes asymptotically when sample size increases.

Calibration. MIMIC model output gives only shadow economy index that shows dynamics of latent variable. In order to obtain the size of the shadow economy (share of shadow economy in total GDP) it is needed to use the calibration procedure. The main drawback is related with the option of the starting values since it has great impact on the calculations. In terms of level calibration, MIMIC is based on external CDM estimates. As a result, some drawbacks from CDM (e.g. controversial assumption regarding the equality of the velocity of money in the formal and informal economy) reflect on MIMIC model as well.

2.3. Structured hybrid model

Econometric estimates of the shadow economy could be explained by a different methods. Although CDM and MIMIC are dominant approaches in the literature, these models have certain weak points that were mentioned in the previous sections. For that reason, this subsection is aimed to analyze new approach in order to minimize shortcomings of both models. By combining CDM and MIMIC models, it is expected that more reliable results may be received.

During literature review, paper that introduces a new approach for measuring shadow economy was analyzed (see [15]). It is a very recent article written by P.Dybka et al. and published in 2019. Authors briefly review weak spots of the currency demand approach and MIMIC model and propose an unified statistical model called structured hybrid method for the estimation of the shadow economy.

The key point in SHM is that much attention is paid on the mean and variance of the shadow economy obtained form CDM. It is presumed that this information is treated as given and it is going to be used into a maximization of the restricted full information maximum likelihood function. The application of SHM mainly focuses on identification problem that arises in MIMIC since mistakes of an an identification may cause misleading results. Internal and consistent identification is achieved by providing vectors of means and variances instead of typically restricting a single element of λ in (5) or γ in (4). As a result, this scheme is characterized as "reverse standardization" in the paper.

Thus, authors highlight 3 principal steps in modeling SHM:

1. estimation of the panel version of the extended currency demand model with an inclusion of additional variable of electronic payment system;
2. extraction of vectors of a panel specific means and variances from the CDM;
3. estimation of the MIMIC by using the restricted full information maximum likelihood function and taking into account of an information of means and variances from CDM.

After consultations with Professor A. Torój (corresponding author of an analyzed article), in this work it was decided to use the data of Lithuania only (not a data set structured as a panel) since the bright side of the single country standpoint is the usage of more variables arguing that country specificity is accounted in a better way. Also, similar information is highlighted in the article as a statement that separate model may better explain the variation in the dependent variable for a certain country due to specificity of predictors, opportunity of taking longer time series and avoidance of an outliers among the fixed effect. However, the cost of a single country approach is potentially higher standard errors of the estimates and the loss of degrees of freedom (lesser amount of observations).

From a mathematical side, similarly as MIMIC, structured hybrid model (in a form of a single country and including a constant of the mean of the shadow economy) can be written down into 2 equations (7) and (8):

$$\widetilde{\eta}_t = \mu_{\widetilde{\eta}} + \gamma^\top X_t + \zeta_t \tag{7}$$

$$\widetilde{y}_t = \lambda \widetilde{\eta}_t + \varepsilon_t \tag{8}$$

where $\widetilde{\eta}_t$ is a latent variable of a structured hybrid model in time t and $\mu_{\widetilde{\eta}}$ is an expected value of $\widetilde{\eta}_t$.

Then, following the procedure presented in [15], diagonal variance-covariance Σ matrix is derived and a restricted full-information maximum likelihood procedure is performed where non-negativity constraints on error variances are included. If variances of measurement errors are non-positive, then a likelihood is penalized by a large (in absolute terms) negative value (see Figure 3). Technically, modified RFIML provides a superiority of blocking negative values of predictor-specific variances of the errors as the diagonal elements of Σ matrix. In the contrary, positive sign of all the variances is not ensured in the estimation of standard or unrestricted MIMIC ("lavaan" package). Hence, this could be a warning of misspecification, herewith discrepancy among causes and consequences of the model. More details of the estimation procedure of structured hybrid model could be found in the article [15] on pages 21-22.

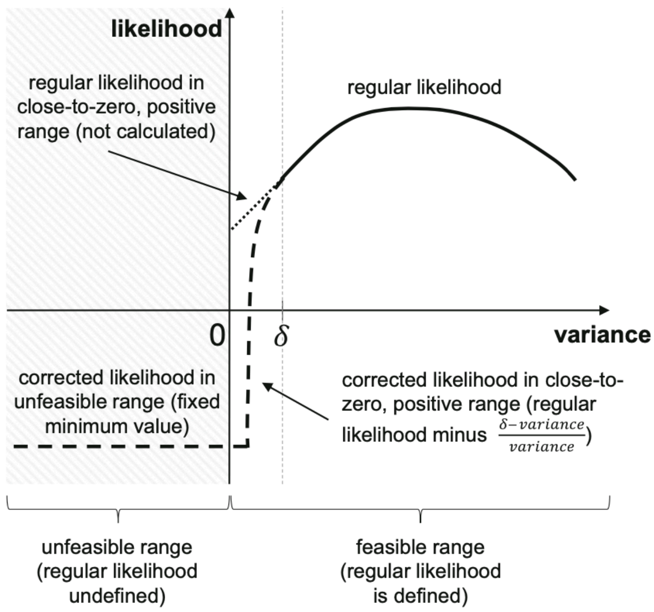

Figure 3. Modification of the likelihood in restricted full information maximum likelihood estimation. Source: [15].

To sum up, econometrically, a novel approach of SHM is considered as way more superior in comparison of unrestricted MIMIC.

3. Empirical investigation

In this section, a brief overview of modeling variables, some data's characteristics and the results of methods used to estimate the size of the shadow economy is presented. All calculations was performed using R computing environment [54].

3.1. Data description

The data used for the analysis was extracted from various sources: the database of Lithuanian Department of Statistics (LDS), Bank of Lithuania (BL), Eurostat, the Organisation for Economic Co-operation and Development (OECD), Federal Reserve Economic Data (FRED), European Central Bank (ECB) (see Table 1). Based on data availability, quarterly data covering the period from 1999 Q4 to 2020 Q2 is analyzed. The advantage of quarterly data comparing to annual data is higher number of observations (in our case 83 observations), thus, the time series are longer. In addition, year may be treated as an excessively long period of time, as the responses of the variables to the effects may unfold over a shorter period than a year. The first look of the modeled variables is provided in the appendix (see Figure A).

Table 1. List of variables.

Indicator	Short name	Measurement	Source
Tax burden	TAXB	% of GDP	ECB
Budget tax revenue	TAXTOT	Euro (Millions)	LDS
Gross domestic product	GDP	Euro (Millions)	Eurostat
GDP per capita	GDPCAP	Euro	LDS
Short-run interest rate	R	%	OECD
Unemployment rate	UR	%	LDS
Male unemployment rate	MUNEMP	%	LDS
Monthly salary (bruto)	W	Euro	LDS
Government deficit(-) or surplus(+)	GDS	% of GDP	ECB
Residential property prices	RPP	Index 2010=100	FRED
Harmonised index of consumer prices	CPI	%	Eurostat
Money ratio	MON_RATIO	Euro (Millions)	Derived [1]
Money supply M2	M2	Euro (Millions)	BL

[1] Note: derived from the data of cash outside banks and M2 from BL.

Interesting and unusual (in the context of other authors' work) indicator is residential property prices for Lithuania (the coverage includes all types of new and existing dwellings in the whole country). The data was extracted from FRED database. Later it will be shown that higher residential property prices stimulates shadow activities in the country. Also, in order to build currency demand model, it was needed to derive money ratio variable from cash and M2 money supply (both indicators' data was taken from the Bank of Lithuania). More information on all the considered explanatory variables and their possible impact on the size of the informal economy could be found in Subsubsection 1.1.2.

Besides, in this study, several other important aspects of the data collection may also be distinguished:

- **Seasonality.** Macroeconomic indicators (GDP, unemployment, wages, etc.) might have seasonality, especially when having more frequent data than annual. Often, due to seasonal fluctuations, time series are non-stationary. Some indicators were taken from databases as seasonally adjusted. For other variables the seasonal component was found by using Loess smoothing (see Figure B in the appendix). Loess smoothing was applied to TAXTOT, GDP, GDPCAP, R, UR, MUNEMP and W, since these variables were found to be the worst seasoned. Hence, it is assumed that the time series have no seasonal components or they are at least partially eliminated.
- **Money supply changes in 2015.** Lithuania joined the Eurozone by adopting the euro on 1 January 2015. The methodology for calculating some indicators of the money supply changed after Lithuania became a member of the euro area, therefore it has to be considered the comparison of money supply data before 2015 and after 2015.
- **Wage changes in 2019.** It should be noted that in 2019, very high growth of wages was recorded in Lithuania (40 % change com- pared to 2018). Such salary growth is extremely unusual and exceptional due to the changes in tax system and income taxation. For these reasons, the wage values by 2019 need to be recalculated and indexed 1,289 times.

3.2. Tests of a unit root and stationarity

In order to evade problem of spurious regression, stationarity of variables must be ensured. For that reason, testing of time series for a unit root properties is required. Widely known tests are going to be used:

1. **Dickey-Fuller test (ADF).** Firstly, variables in levels are tested. If the considered indicators have a unit root in levels, then first differences of logarithmic variables (logarithm is taken if variable may not contain negative values) are tested. In general, the stationary time series has a time-constant variance and mean. However, in the long-run, most of macroeconomic variables (GDP, wages, inflation, etc.) tend to grow and have an explicit non-zero mean.

Hence, it was decided to perform ADF test with drift type and to choose specification of the Dickey-Fuller test with constant. If ADF test-statistics is more negative than the respective τ critical value, then the null hypothesis of the existence of a unit root of the time series is rejected. Also, it must be noted that a unit root tests were applied in R programming environment using the package "urca" since it gives more information and control over test than using package "tseries".

2. **Kwiatkowski-Phillips-Schmidt-Shin test (KPSS).** Findings about stationarity may be supported by KPSS test. Here, the null hypothesis states that the series is stationary and cannot be rejected if test-statistics is less than critical value).
3. **Phillips-Perron test (PP).** Null hypothesis of Phillips-Perron test is that time series has a unit root.

Table 2 presents the results of the unit root tests for all series in levels and at first differences.

Table 2. Test-statistics of ADF, KPSS, PP.

	ADF		KPSS		PP	
	In level	$d = 1$	In level	$d = 1$	In level	$d = 1$
TAXB	-2.29	-3.57	0.40	0.46	-2.74	-5.10
TAXTOT	-0.03	-4.59	1.81	0.08	-0.25	-9.86
GDP	1.03	-3.16	2.08	0.39	-0.75	-5.85
GDPCAP	-0.59	-3.58	2.10	0.22	-0.08	-5.12
R	-2.03	-6.78	1.41	0.39	-6.04	-11.6
UR	-2.16	-2.78	0.49	0.10	-1.65	-4.05
MUNEMP	-2.16	-3.38	0.36	0.10	-1.77	-4.80
W	0.53	-2.56	2.03	0.12	1.86	-4.28
GDS	-1.68	-5.21	0.33	0.08	-1.74	-7.72
RPP	-1.15	-5.78	1.51	0.31	-1.04	-7.03
CPI	-0.11	-4.26	2.10	0.16	-0.14	-6.28
MON_RATIO	0.33	-6.10	1.52	0.15	0.32	-9.88
M2	3.64	-4.78	2.11	0.60	3.79	-8.99
Critical values (5 %)	-2.89	-2.89	0.46	0.46	-2.90	-2.90

Note: $d = 1$ indicates first difference.

To summarize, Table 2 implicates that there is no problem with a unit root for variables in first differences. Moreover, findings about stationarity of indicators in first differences were supported by KPSS test. Thus, results of ADF, KPSS, PP showed that the order of integration of all indicators is $I(1)$.

3.3. Modeling results of CDM

Further, it is going to be presented the results derived by using the approach of currency demand model.

Variables were non-stationary in levels, but they become stationary after first differences. Since all series with a unit root were said to be integrated of order one ($I(1)$), it is needed to check if considered variables are cointegrated. Therefore, firstly, the long-run currency demand equation (2) should be estimated where dependent variable is MON_RATIO. As mentioned earlier, the level of taxation is of utmost importance in modeling CDM. Consequently, a great attention must be paid on this indicator and a more detailed information about this driver must be provided. First of all, attempts were made with using the variable TAXB from the Table 2 in the modeling. Nevertheless, it was observed that better results were obtained with own calculated tax indicator that is the tax revenue to GDP ratio. The new predictor TAXB_NEW was derived from TAXTOT and modified GDP. Instead of using data of GDP from Eurostat database, it was decided to take into account of the trend component by using Hodrick-Prescott filter (see Figure 4).

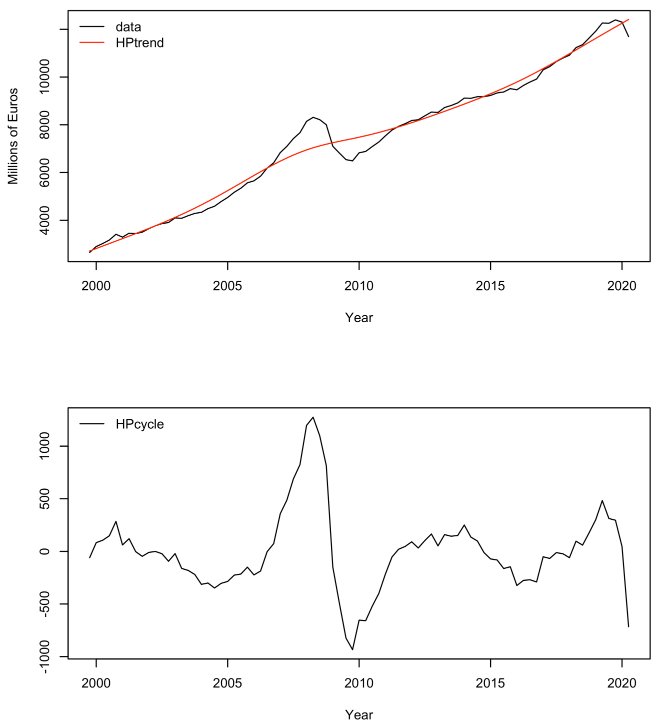

Figure 4. The decomposition of GDP with Hodrick-Prescott filter ($\lambda = 1600$).

Bearing in mind that we model the long-run currency demand, the modification of GDP by trend that indicates the long-term movement of an economic might cause a better reflection of the economic situation of the country in the long-run. As a result, modified GDP (the denominator of variable TAXB_NEW) is as follows:

$$GDP\_MOD_t = \frac{GDP_t}{HPtrend_t}$$

The fit of currency demand model was strongly preferable with the new indicator TAXB_NEW, albeit in economic terms a new variable is similar to TAXB presented by European Central Bank. Hence, in the following analysis it is going to be used:

$$TAXB\_NEW_t = \log\left(\frac{TAXTOT_t}{GDP\_MOD_t}\right)$$

The output of long-run CDM is presented in Table 3. Assumption of no autocorrelation in the residuals is confirmed from ACF plot of the residuals (see Figure C in the appendix). Also, a strong evidence of statistical significance of all coefficients exists.

Table 3. Long-run CDM.

	Estimate	Standard error	t value
$Intercept$	-4.053^{***}	0.841	-4.818
$TAXB\_NEW$	0.301^{**}	0.103	2.920
$MUNEMP$	0.064^{***}	0.014	4.608
R	-0.023^{***}	0.004	-6.468
CPI	0.009^{***}	0.002	4.017
$MUNEMP : CPI$	-0.0008^{***}	0.0001	-6.102

$^{***}p < 0.001, ^{**}p < 0.01$
$R^2 = 0.87$
Adjusted $R^2 = 0.86$

Besides, all signs of the coefficients are logical and interpretable. The particularly important assumption of tax burden as the main reason to engage in illegal activities is confirmed since positive sign of taxation indicator is detected. The male unemployment rate has a positive impact in the long-run (negative coefficient is expected in the short-run) as highlighted in Subsubsection 1.1.2. This driver was selected as it was observed that the male unemployment rates were slightly higher than the female unemployment rates during the analyzed period. Also, the remaining variables are statistically significant and appropriate in economic terms. Moreover, interaction effect of male unemployment rate and inflation increased R^2 of the model. For that reason, interaction term was included in the model additionally.

Moving forward, a cointegration test was performed in order to verify if there is a long-term relationship between the variables. Therefore, the residuals of the long-run estimated model were checked. If the residuals are stationary, model suggests that indicators are cointegrated. To confirm that residuals are stationary we apply Engle-Granger Augmented Dickey-Fuller test for cointegration (EG-ADF test). Test rejected the null hypothesis of nonstationarity of the residuals with a significance level of $\alpha = 0.01$ (test-statistic < critical value respectively $-3.003 < -2.6$). Hence, we could build ECM since we have a strong evidence that the residuals are stationary implying cointegration. The long-run currency demand model (see Table 3) allowed to facilitate procedure of specification of the error correction model. Many combinations of ECM were tested and the results of selected ECM is produced in Table 4. It could be seen that all covariates are statistically significant. Also, drivers have meaningful economic interpretation.

Table 4. ECM.

	Estimate	Standard error	t value
$Intercept$	$-3.55 \cdot 10^{-1}$	$3.109 \cdot 10^{-1}$	-1.143
$\Delta MUNEMP$	$4.84 \cdot 10^{-2*}$	$2.252 \cdot 10^{-2}$	2.150
ΔR	$-3.29 \cdot 10^{-2***}$	$6.772 \cdot 10^{-3}$	-4.859
$\Delta interaction$	$-5.01 \cdot 10^{-4\cdot}$	$2.552 \cdot 10^{-4}$	-1.962
R_{t-1}	$-1.52 \cdot 10^{-2***}$	$2.962 \cdot 10^{-3}$	-5.116
$interaction_{t-1}$	$4.80 \cdot 10^{-5**}$	$1.580 \cdot 10^{-5}$	3.040
$TAXB\_NEW_{t-1}$	$7.45 \cdot 10^{-2\cdot}$	$4.264 \cdot 10^{-2}$	1.747
CPI_{t-1}	$-3.59 \cdot 10^{-3**}$	$1.120 \cdot 10^{-3}$	-3.204
$\log(MON\_RATIO)_{t-1}$	-1.37^{***}	$9.911 \cdot 10^{-2}$	-13.897

$^{***}p < 0.001, ^{**}p < 0.01, ^{*}p < 0.05, ^{\cdot}p < 0.1$

$R^2 = 0.75$

Adjusted $R^2 = 0.73$

Based on the assumption of the tax burden, a positive coefficient of taxes implies an increase in the amount of money in the market (at the same time larger shadow economy) as the tax predictor increases. The negative coefficient of the short-run interest rate represents the cost of holding cash compared to saving. Male unemployment rate is with positive coefficient since this labour market indicator pushes economic agents to operate in a shadow activities in order to increase the level of personal income. Negative impact of harmonised index of consumer prices is explained in

Subsubsection 1.1.2 as well as the confirmation of existence of a long-run significant negative effect of inflation on the demand for money could be found in [26]. The long run effect of specific input could be obtained by dividing the lagged independent indicators by the lagged dependent variable (money ratio) Also, it may be seen that in the ECM even 3 variables (male unemployment rate, short-run interest rate and interaction term of male unemployment rate and inflation) describing the short-term effect were included. The coefficient of the lagged money ratio is treated as the error correction term. Negative coefficient indicates the speed of adjustment and implies a convergence from short-run to long-run. Besides, it should be noted that the speed of adjustment in the model is extremely large since the system adjusts its previous period disequilibrium at the speed of 137 %. In spite of the fact that it is an unusually large correction, [35, 38] claims that the stable coefficient on the error-correction term could be lower than -1, but not be lower than -2. According to [38], the speed of adjustment that lies between -1 and -2 indicates not a monotonic convergence to the equilibrium path, but the error correction process that fluctuates around the long-run value in a dampening manner. Nevertheless, when this process is completed, convergence to the equilibrium path is going to be instant.

Hence, the most appropriate model was selected (bearing in mind the importance of interpretation, assumptions of CDM and model diagnostics). Once more, there was found no autocorrelation in the residuals (see Figure D in the appendix). The Ljung-Box and Box-Pierce tests confirmed that the residuals are white noise (p-value > 0.05 $\implies$ H_0 of no serial correlation could not be rejected). Exactly the same output confirming that there is no evidence for autocorrelation is presented by Durbin-Watson test. Moreover, Table 5 presents results of the performance of other residual diagnostics tests. The Lagrange multiplier (LM) test for autoregressive conditional heteroskedasticity (ARCH) confirms the presence of the squared residuals as a sequence of white noise (residuals are homoscedastic). The normality assumption is validated by Kolmogorov-Smirnov test. Finally, the stationarity of residuals was detected with KPSS test (test-statistic < critical value accordingly 0.08 < 0.74, with a significance level of $\alpha = 0.01$). To conclude, the model's residuals are well-behaved.

Table 5. Residual diagnostics of ECM.

Test	p value
Ljung-Box	0.138
Box-Pierce	0.145
Durbin-Watson	0.348
ARCH	0.242
Kolmogorov-Smirnov	0.335

Additionally, predicted values by ECM (black curve) and the actual values (red curve) could be seen in Figure 5. The predicted values of logarithmic growth rates (in percentage) of money ratio are fairly close to the actual ones and that is a result of quite high value of R^2 (75 % of the variance for a dependent variable could be explained by the model's inputs). To sum up, it may be concluded that ECM presents sufficiently adequate results.

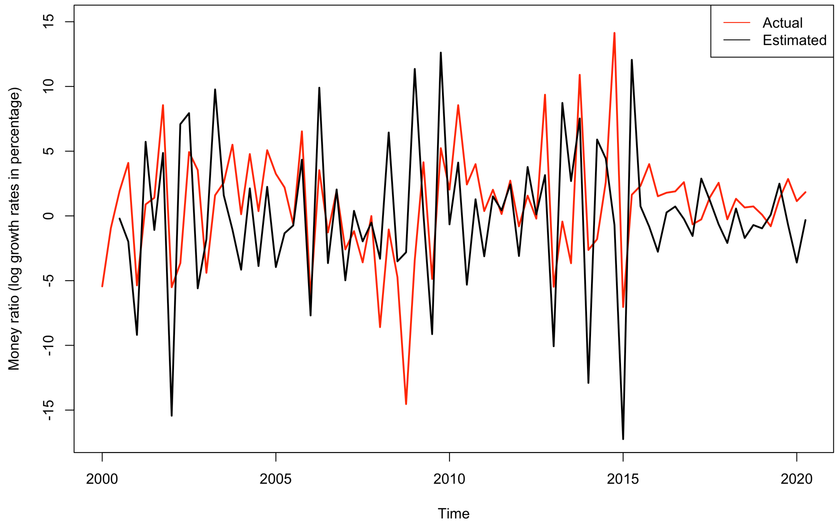

Figure 5. Fit of ECM.

Next, the forecasts of growth rates were antilogarithmized and the level values of money ratio is found with the help of a vector of cumulative product (obtained with $cumprod()$ function). Finally, using methodology described in Subsubsection 2.1.1, the size of the shadow economy as a % of GDP may be determined (see Figure 6). Quarterly shadow economy estimates are presented in the appendix (Table 1).

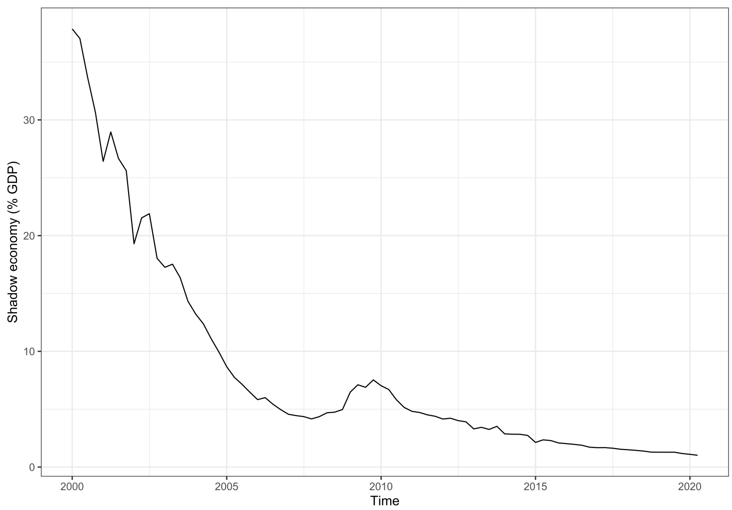

Figure 6. The size of the shadow economy as % of GDP (CDM).

3.4. Modeling results of MIMIC

There is a large body of literature on the possible causes and indicators of the shadow economy. The set of causes and indicators (see Subsubsection 1.1.2) that is going to be used in MIMIC modeling was selected according to the availability of the data and other authors' recommendations in their researches. In most of articles it was found that the development of the informal economy is affected by taxes, unemployment and wages while a changes in the size of the shadow economy is reflected through a changes of the monetary aggregates and gross domestic product.

Thereafter, R package "lavaan" [41] was used in order to build MIMIC models. The stationarity of the drivers was ensured by employing first differences (see Table 2). In the process of modeling MIMIC, various combinations of causes and consequences were tested, the interpretability of coefficients were analyzed and statistically insignificant variables were eliminated. At the same time, models were compared according to criteria presented in Subsubsection 2.2.2. Results of the most adequate MIMIC is given in the Table 6 and the "Lavaan" output of the MIMIC is shown in Figure E).

Table 6. The output of MIMIC.

Causes	Estimate
ΔGDS	1.00
$\Delta \log(TAXTOT)$	0.12^{**}
	(0.039)
ΔUR	-1.46^{**}
	(0.271)
$\Delta \log(RPP)$	0.12^{**}
	(0.039)
$\Delta \log(W)$	0.03^{***}
	(0.008)
Indicators	Estimate
$\Delta \log(GDPCAP)$	0.08^{***}
	(0.02)
$\Delta \log(M2)$	0.04^{*}
	(0.02)

$^{***}p < 0.001, ^{**}p < 0.01, ^{*}p < 0.05$

Note: standard errors in parentheses.

As in CDM, estimates of variables in MIMIC have meaningful economic signs. The MIMIC model indicates that the higher level of taxation, residential property prices and wages, the more economic agents operate in the shadow economy market in Lithuania. Meanwhile the higher unemployment rate the less likely is participation in the shadow economy. In this case, as explained in [5], income effect exceeds the substitution effect indicating countercyclical movement of the shadow economy with the level of unemployment rate. In order to estimate MIMIC model, predictor government deficit(-) or surplus(+) was chosen for normalization to have a unit coefficient regarding the standard assumption of $\gamma_1 = 1$.

Furthermore, GDP per capita and money supply M2 are treated as indicators the most appropriately reflecting the extent of the shadow economy. Thus, a classical MIMIC rule of identification [31] (unobserved variable should cause at least two indicators and unobserved variable should be caused by at least one exogenous variable) is followed.

The correctness of the MIMIC model was checked by the fit indices (see Table 7). Chi-square test statistic, SRMR, RMSEA, TLI, CFI confirmed that the model is properly specified and predictions match the actual data (see Subsubsection 2.2.2).

Table 7. Diagnostics of MIMIC.

Goodness-of-fit statistics	
Degrees of freedom	13
p-value (Chi-square)	0.143
SRMR	0.064
RMSEA	0.039
TLI	0.946
CFI	0.966

Following the approach about the analysis of moving sums of residuals presented in [11, 59], the stability of the parameters of the MIMIC model was investigated in the context of the theory of MOSUM processes when structural changes are observed in the data. Instead of holding the sum of all residuals up to a certain time, the empirical fluctuation processes hold the sum of a fixed number of residuals in a data window that size is controlled by the moving over the whole sample period bandwidth parameter $h \in (0,1)$. More information of mathematical principles of MOSUM test could be detected in [60]. Accordingly, all the obtained graphs (see Figure 7) of the MOSUM processes do not cross the critical limits as the sample size increases. Thus, it can be stated that all the parameters are stable or in other terms they have no structural changes.

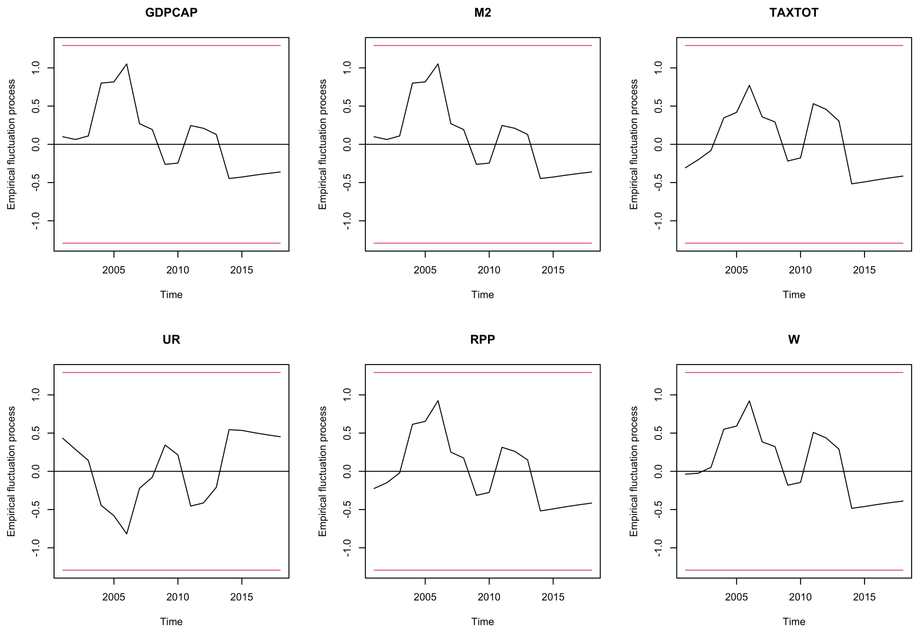

Figure 7. Parameters' stability verification (MOSUM processes).

As was mentioned earlier, MIMIC model provides only shadow economy index that shows dynamics of latent variable (see Figure 8). Hence, further calculations may be performed on the basis of the shadow economy's trajectory.

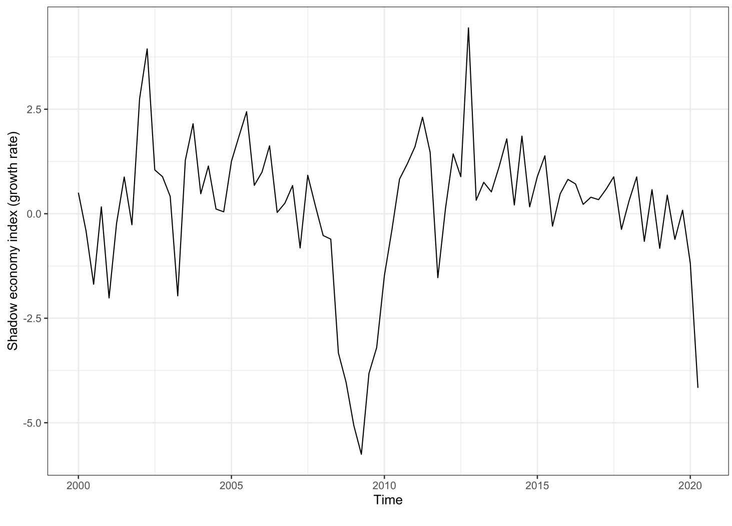

Figure 8. Shadow economy index.

With a reference to CDM, shadow economy in 2000 Q1 was 37.86 % of GDP. As a result, this value is treated as the base value that is going to be used in the transformation of the latent variable into interpretable levels (absolute values) of the shadow economy. The share of shadow economy in total GDP is obtained by the calibration procedure. In a nutshell, performed benchmark method (see [44]) may be described as follows:

$$\hat{\eta}_t = \frac{\widetilde{\eta}_t \cdot \eta^*_{2000Q1}}{\widetilde{\eta}_{2000Q1}} \quad \text{for } t = 2000, \ldots, 2020$$

where $\hat{\eta}_t$ is an absolute values of the shadow economy in time t, $\widetilde{\eta}_t$ is a MIMIC index in time t, $\widetilde{\eta}_{2000Q1}$ is a MIMIC index in 2000 Q1, η^*_{2000Q1} is the base value in 2000 Q1 (in our case it is 59.46 %).

Thus, finally, the size of the shadow economy in total GDP could be calculated (the output is shown in Figure 9).

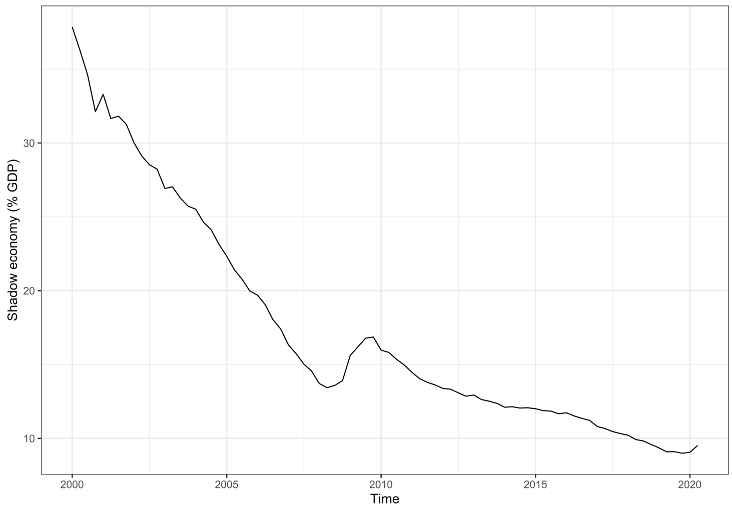

Figure 9. The size of the shadow economy as % of GDP (MIMIC).

Quarterly shadow economy estimates of the MIMIC are presented in the appendix (see Table 2).

3.5. Modeling results of SHM

In this section, the output of joint CDM-MIMIC is presented. For comparison purposes, it was decided to take results (mean and standard deviation of dependent variable) of an estimated CDM (see Subsection 3.3) and then build an advanced SHM based on previously presented novel restricted full-information maximum likelihood procedure. Thereby, mean and standard deviation of the shadow economy obtained from CDM could be treated as a linkage between CDM and unrestricted MIMIC. Also, the causes and indicators used in regular MIMIC with "lavaan" package (see Subsection 3.4) were taking into account while modeling SHM.

Further, the key differences (comparing with [15]) in modeling SHM will be discussed. First of all, as mentioned earlier, it was decided to apply SHM on Lithuanian data only. Secondly, there should be noted that in a paper a variable of electronic payment system is included in a currency demand model. Nevertheless, the majority of authors do not include payment card system variable since it do not meet the theory proposed by founder of a model V. Tanzi (1). Moreover, keeping in mind that starting point of modeling is 2000 (that data of Lithuania is likely to be unreliable) it was decided to stay at the cash-related CDM model. Thirdly, authors of the article use money aggregate M1 instead of M2. Again, this standpoint contradicts to the approach introduced by V. Tanzi. In the case of the data of Lithuania, both money aggregates were tested (M1 and M2), however better statistical inference were obtained with a broader definition of money involving saving deposits. Although M1 is more liquid than M2, not including savings deposits could lead to underestimation of the size of the shadow economy since this type of money does not require a great effort to withdraw and spend (e.g. a check cannot be written directly, but a certain amount of money can be easily withdrawn at an automated teller machine or bank). Moreover, it is known that often CDM estimates a smaller size of the shadow economy than it actually is. Hence, there should be no problem with alleged overestimation. Additionally, the choice of M2 over M1 may be justified by the evidence of B. Friedman that showed unstable money demand when targeting M1 [22].

Next, an outcome of a maximization of RFIML (this restricted full information maximum likelihood function was written in R) is going to be presented. As stressed before, "lavaan" solution does not handle the negative variance problem. Therefore, constraints that variance has to be positive have been imposed. Nevertheless, readily available "lavaan" package was first used for getting initial values of an unconstrained MIMIC. Then, a rate of downscaling initial γ (the best value was found to be 0.788) was used for the correction of starting values since variances were not positive.

Finally, maximization procedure was executed taking into account the mean (8.246) and standard deviation of the shadow economy computed from CDM. Statistical inferences of SHM is provided in Table 8. Parameter estimates were obtained after 2266 function evaluations. The interpretation of causes and indicators and herewith the signs of the coefficients are the same as in estimated MIMIC model (see Table 6). Also, structured hybrid model indicates no problem of variances' negativity. In addition, it should be stressed that the standard errors were obtained as the square roots of the inverse Hessian matrix's diagonal elements. As explained in the analyzed paper [15], not all standard errors may be calculated since constraint may affect concavity of a likelihood function (see Figure 3). As a consequence, diagonal elements in inverse Hessian matrix become negative and the roots cannot be obtained. Nonetheless, in the opposite situation, other coefficients are said to have statistical significance as Table 8 indicates. Furthermore, as discussed earlier, normalization condition is not required in SHM. Hence, in contrast to the unrestricted MIMIC model, there is no need to normalize either cause or indicator to have a unit coefficient.

Table 8. Results from the SHM estimation.

Causes	Estimate	Standard error	t-stat	p-value
ΔGDS	0.102	NA	NA	NA
$\Delta\log(TAXTOT)$	0.743	NA	NA	NA
ΔUR	-0.180	0.000	-3297449.99	0.000
$\Delta\log(RPP)$	1.79	0.000	2119391.22	0.000
$\Delta\log(W)$	1.716	NA	NA	NA
Indicators	Estimate	Standard error	t-stat	p-value
$\Delta\log(GDPCAP)$	0.061	0.030	2.085	0.041
$\Delta\log(M2)$	0.054	0.027	2.013	0.048
Variances	Estimate			
$\Delta\log(GDPCAP)$	0.006			
$\Delta\log(M2)$	0.005			

Note: NA - not available (explanation in a text).

Eventually, based on SHM estimates, dynamics of the shadow economy may be seen in Figure 10.

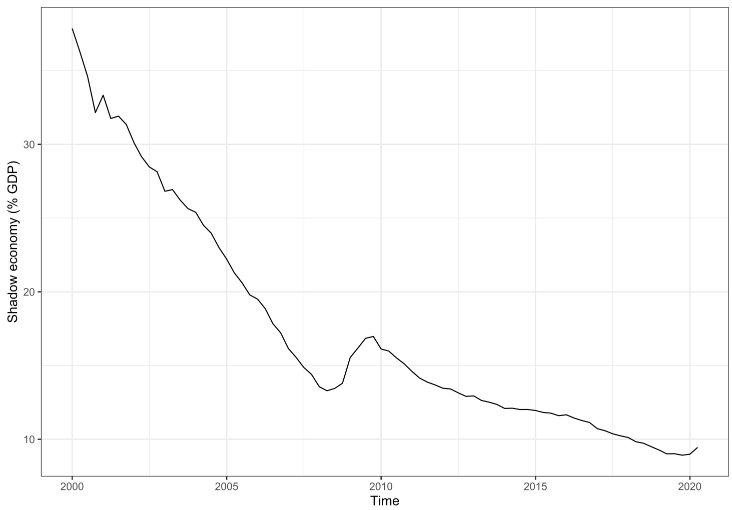

Figure 10. The size of the shadow economy as % of GDP (SHM).

Quarterly shadow economy estimates of the SHM are presented in the appendix (see Table 3). It is detected that the quarterly shadow economy's sizes are extremely similar to that of the MIMIC model.

3.6. Comparison of results and different estimation methods

Although MIMIC and SHM have evaluated sufficiently large shadow economy size in comparison of CDM, all three models have shown the same trajectory of the informal economy (see Figure 11).

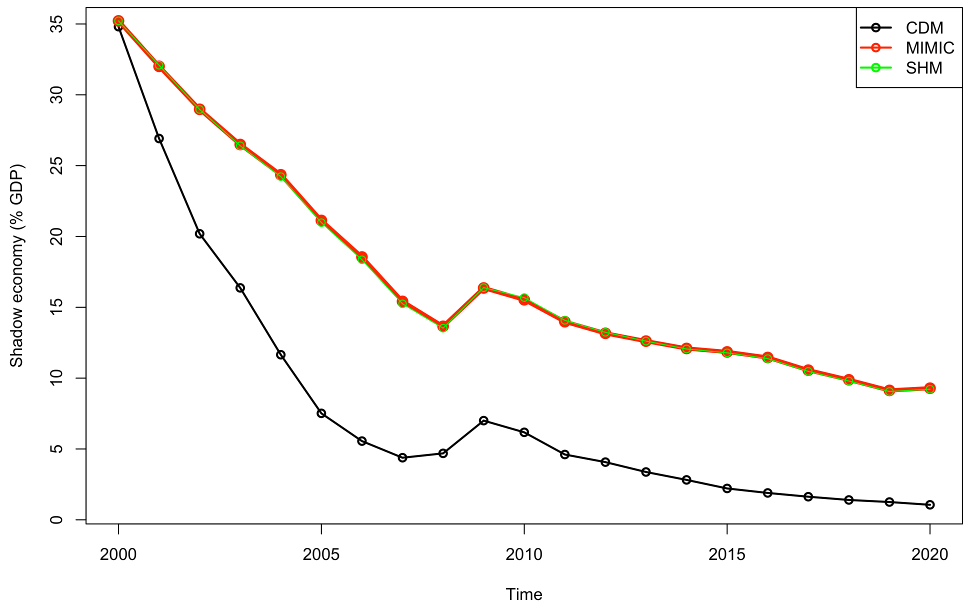

Figure 11. Paths of the shadow economy in Lithuania (CDM, MIMIC and SHM output).

Besides, it might also be noted that an average annual values of the shadow economy are tended to shrink over time in Lithuania (except recession period in 2008-2009). Also, as Table 9 indicates, based on MIMIC and SHM Lithuania experienced a small increase in the size of an informal economy during 2019 and 2020. This can be treated as a signal for the potential growth of the shadow economy in the future since current global pandemic of COVID-19 should positively affect the market of the shadow. Thus, immediate responses and changes of policy carrying out by policy makers may be considered by paying a great attention on the effects of causes determined in the models (inflation, unemployment, wages, etc.). Nevertheless, general tendency of a reduction of the shadow activities over analyzed period of 2000-2020 may be explained by the strategy implemented of the government of Lithuania. For example, in order to reduce the shadow economy government gave 6 months to pay the "forgotten" taxes without any penalties. Another measure is related with incentives to the fair entrepreneurship (benefits, better conditions, access to bank financing, etc.). Also, better tax administration and improving economic outlook in general may also have had an impact on the decrease of the shadow economy's activities.

All in all, the correspondence between the results of the models and economic events seems a convincing justification to state that by the help of econometric modeling the development of the shadow economy in Lithuania has been identified correctly.

Table 9. The average annual values of the shadow economy (% GDP).

	CDM	MIMIC	SHM
2000	34.809	35.218	35.222
2001	26.915	32.010	32.088
2002	20.190	28.976	28.966
2003	16.371	26.491	26.403
2004	11.652	24.356	24.223
2005	7.514	21.130	20.975
2006	5.556	18.551	18.356
2007	4.382	15.416	15.245
2008	4.690	13.657	13.525
2009	7.001	16.364	16.387
2010	6.175	15.532	15.687
2011	4.609	13.983	14.080
2012	4.072	13.164	13.233
2013	3.375	12.613	12.614
2014	2.817	12.095	12.059
2015	2.213	11.850	11.786
2016	1.896	11.457	11.378
2017	1.630	10.563	10.477
2018	1.401	9.882	9.796
2019	1.255	9.134	9.056
2020*	1.059	9.289	9.219

* Note: average of Q1 and Q2 is presented.

Moreover, as discussed earlier, each estimation procedure of CDM, MIMIC or SHM has certain pros and cons. Although CDM seems to underestimate the size of the shadow (since 2016 extremely low values are recorded) an outcome is crucial as a calibration instrument for modeling MIMIC and useful as an auxiliary information (mean and standard deviation) for modeling SHM. Potential reason for the underestimation might be the change in the data of a cash in circulation. Some adjustments were done based on a share in the ECB capital since Lithuania's accession to the Eurozone in 2015. Basically, underestimation has revealed a crucial problem related to the limitations of the data. In other words, the usage of a proxies based on a shares in the ECB capital is quite problematic. The capital share approach means that the value of currency in circulation is estimated on the whole Eurozone level and then for each country it is estimated by multiplication by the share of a given country in the ECB capital. As a result, it does not accurately predict the amount of currency in circulation on the country level.

Comparing MIMIC and SHM, the results do not much differ in economic terms. However, SHM can be treated as a more advanced econometric tool for modeling shadow economy where essential issues of identification and negative variances are solved. A weak point of SHM is related to the contribution from a MIMIC part. When additionally using MIMIC after CDM only 1-2 % (approximately) extra variation of the shadow economy is obtained (A. Torój, personal communication, January 4, 2021). Thus, there is a consideration if a MIMIC part of SHM is a redundant step in the approach of SHM. As a result, in a subsequent work [16] only CDM is modeled.

Conclusions

In this thesis, three models (currency demand, multiple indicators-multiple causes and novel structured hybrid model) were constructed for an identification of the extent of the shadow economy in Lithuania over the period for 2000 to 2020. Although informal economy is considered to be a latent variable, structural equation models cannot be treated as the only one classical instrument of the estimation since calibration and additional information of the dependent variable's mean and variance are needed for obtainment of an interpretable form of an unobservable indicator. In general, according to all models' outcome, a very similar trajectory of the shadow economy and a tendency of a shrinkage have been identified. Regardless, MIMIC-based and SHM-based estimates conditioned higher values of a variable of interest. Over the period 2000-2008 and 2009-2019, Lithuania experienced a decline in the size of informal economy while increase was recorded during 2008-2009 and 2019-2020.

Government budget balance, the level of taxation, unemployment rate, short-run interest rate, harmonised index of consumer prices, residential property prices and average earnings were found to be key determinants of the shadow economy market while GDP per capita and money aggregate M2 were considered as indicators.

Also, it should be stressed that there is no universal definition of the shadow, at the same time, the most accurate method of the estimation of the shadow economy might not be singled out. Nevertheless, in this work, SHM is preferred due to more explicit presentation technique (in a terms of econometric language), restricted estimation procedure and improvements in an identification strategy.

All in all, it could be stated that the aim of this master thesis is achieved. Especially, nowadays, when the future is surrounded by extreme uncertainty of COVID-19 crisis, it may be expected the growth of the shadow activities. As a result, there should be a need for a research covering econometric estimates of the informal economy. Whereas evaluation of the shadow economy is not only the econometricians' target, it is believed that ideas presented in this master thesis will inspire to analyze this complex economic phenomenon and its impact on the country's formal economy.

Furthermore, this work could be extended by including panel data set and building fixed effects panel model in a context of CDM assumptions since certain inputs may have little or no "within" variance (i.e. in a single country case), but they might vary considerably in the "between" dimension (i.e. between countries). If this is the case, lower standard error of the estimates should be obtained.

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Appendix Nr. 1. Graphical analysis of the modeling variables

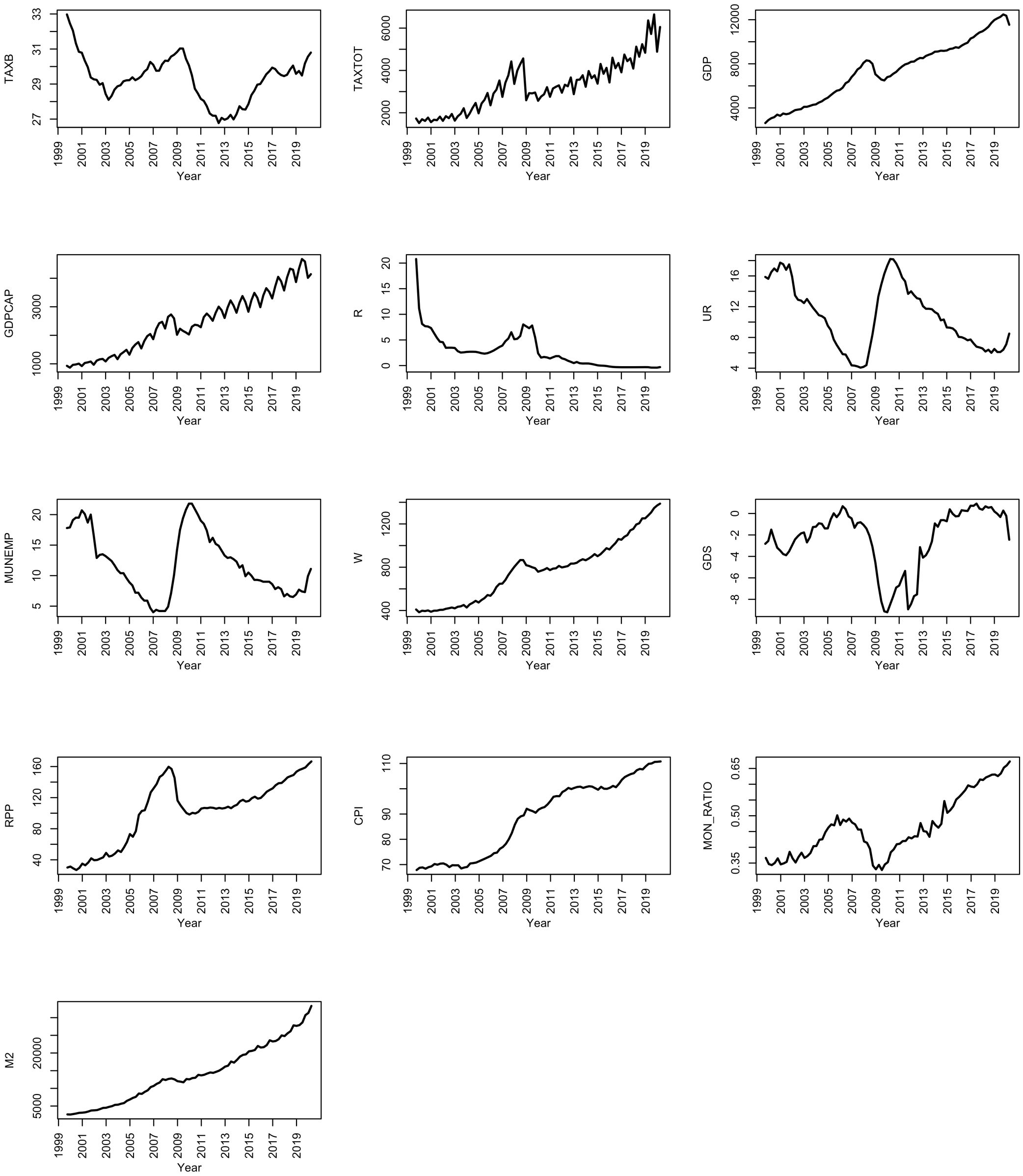

Figure A. Inputs.

Appendix Nr. 2. Seasonally adjusted data

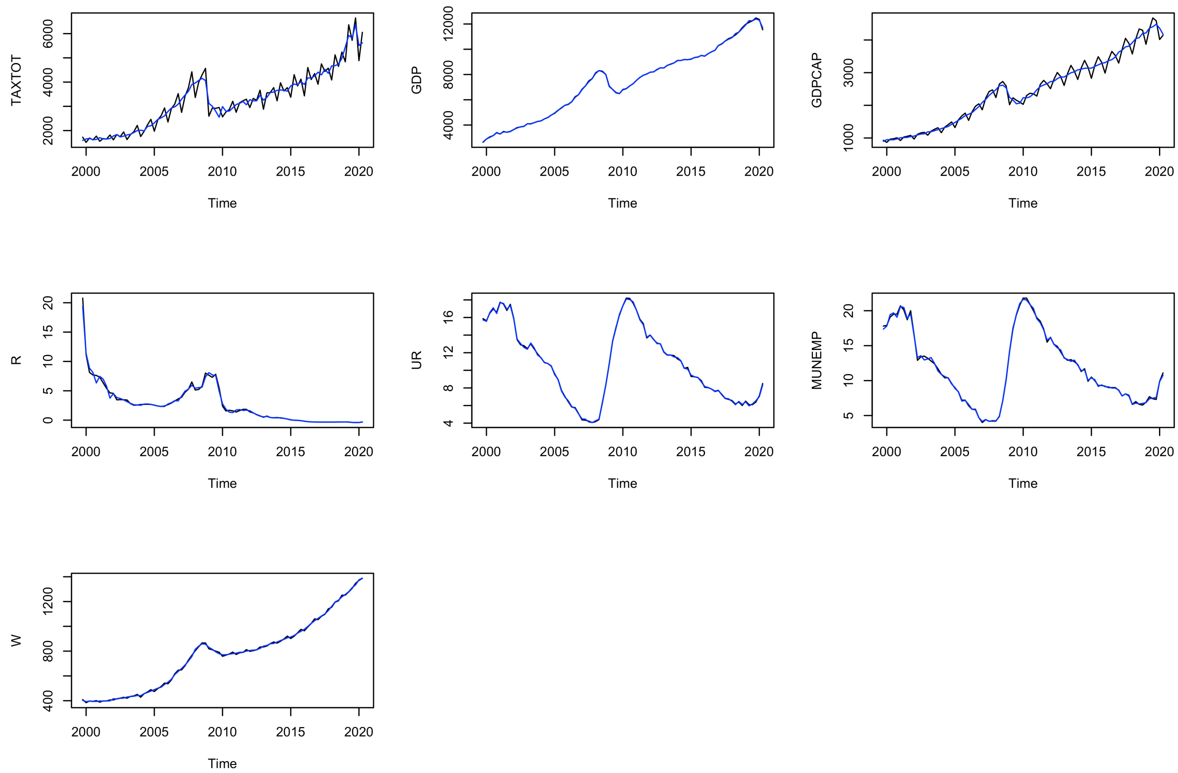

Figure B. Seasonally adjusted data.

Appendix Nr. 3. Residual diagnostics

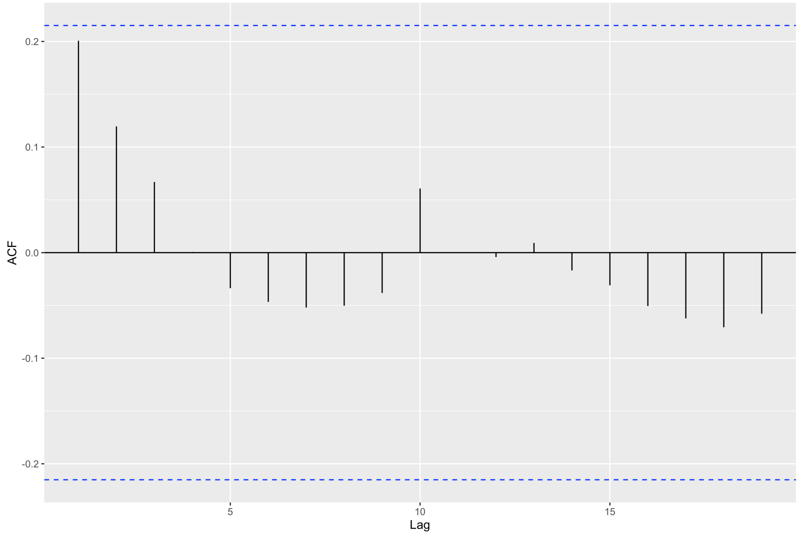

Figure C. ACF plot of long-run CDM model residuals.

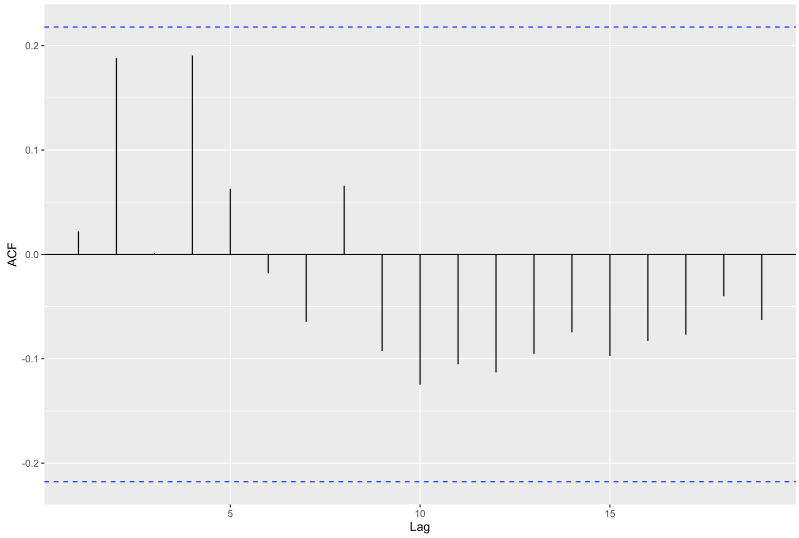

Figure D. ACF plot of ECM model residuals.

Appendix Nr. 4. Shadow economy estimates - CDM

Table 1. Shadow economy % GDP (CDM).

	Q1	Q2	Q3	Q4
2000	37.86	37.03	33.68	30.66
2001	26.42	28.96	26.67	25.61
2002	19.30	21.53	21.89	18.04
2003	17.26	17.53	16.36	14.33
2004	13.21	12.37	11.09	9.94
2005	8.67	7.76	7.15	6.47
2006	5.83	6.00	5.43	4.97
2007	4.56	4.45	4.36	4.16
2008	4.36	4.69	4.74	4.97
2009	6.47	7.11	6.89	7.53
2010	7.03	6.71	5.81	5.15
2011	4.81	4.72	4.51	4.39
2012	4.15	4.22	4.01	3.91
2013	3.30	3.43	3.25	3.52
2014	2.87	2.84	2.83	2.73
2015	2.13	2.35	2.29	2.08
2016	2.02	1.96	1.89	1.71
2017	1.67	1.68	1.63	1.54
2018	1.49	1.44	1.38	1.29
2019	1.28	1.28	1.28	1.17
2020	1.10	1.02		

Appendix Nr. 5. MIMIC model

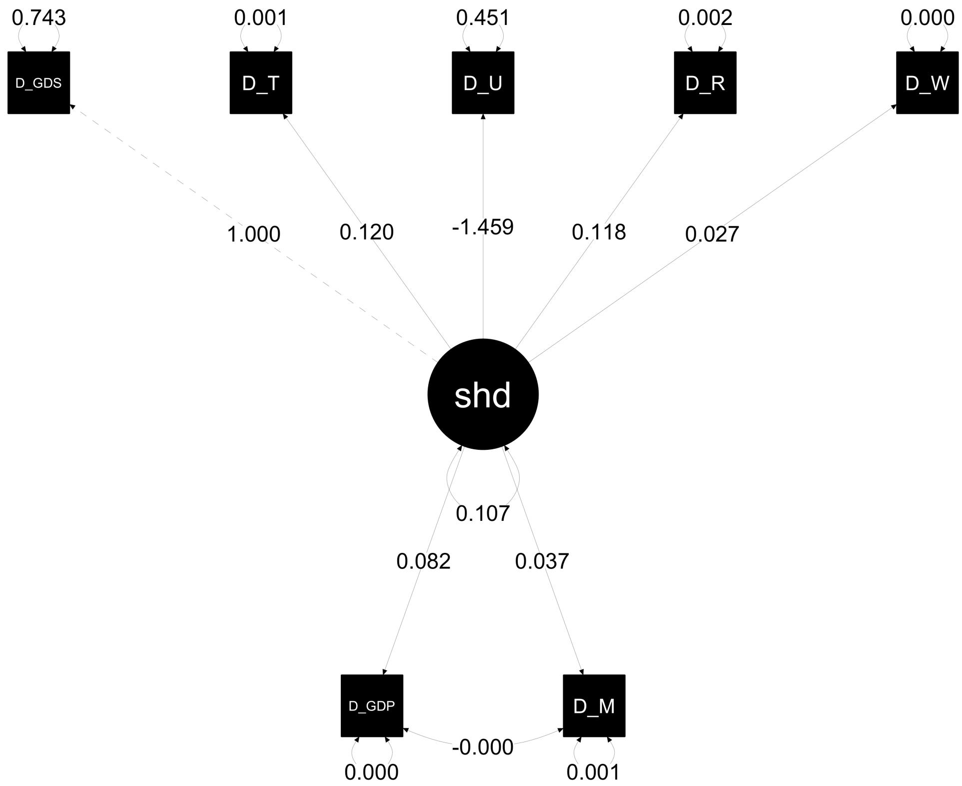

Figure E. "Lavaan" output.

Appendix Nr. 6. Shadow economy estimates - MIMIC

Table 2. Shadow economy % GDP (MIMIC).

	Q1	Q2	Q3	Q4
2000	37.86	36.29	34.61	32.12
2001	33.30	31.66	31.81	31.27
2002	30.01	29.13	28.53	28.23
2003	26.92	27.03	26.28	25.73
2004	25.51	24.64	24.13	23.14
2005	22.34	21.42	20.78	19.98
2006	19.69	19.06	18.04	17.41
2007	16.33	15.74	15.04	14.56
2008	13.72	13.43	13.58	13.90
2009	15.62	16.19	16.78	16.86
2010	15.98	15.82	15.36	14.97
2011	14.47	14.04	13.80	13.62
2012	13.38	13.33	13.09	12.86
2013	12.93	12.63	12.52	12.37
2014	12.11	12.14	12.05	12.07
2015	12.01	11.88	11.85	11.67
2016	11.73	11.52	11.36	11.22
2017	10.80	10.66	10.46	10.33
2018	10.21	9.92	9.82	9.58
2019	9.36	9.09	9.10	8.99
2020	9.06	9.52		

Appendix Nr. 7. Shadow economy estimates - SHM

Table 3. Shadow economy % GDP (SHM).

	Q1	Q2	Q3	Q4
2000	37.86	36.27	34.60	32.15
2001	33.33	31.75	31.91	31.35
2002	30.10	29.15	28.47	28.14
2003	26.82	26.93	26.22	25.64
2004	25.39	24.51	23.98	23.01
2005	22.21	21.29	20.62	19.79
2006	19.50	18.86	17.84	17.22
2007	16.15	15.56	14.88	14.40
2008	13.56	13.29	13.45	13.80
2009	15.55	16.18	16.84	16.97
2010	16.12	15.99	15.52	15.12
2011	14.60	14.15	13.88	13.69
2012	13.46	13.41	13.15	12.91
2013	12.94	12.64	12.52	12.36
2014	12.09	12.11	12.02	12.02
2015	11.95	11.82	11.77	11.60
2016	11.66	11.44	11.77	11.14
2017	10.72	10.58	10.37	10.23
2018	10.12	9.84	9.73	9.50
2019	9.27	9.01	9.02	8.92
2020	8.99	9.45		