VILNIUS UNIVERSITY
FACULTY OF MATHEMATICS AND INFORMATICS
MODELLING AND DATA ANALYSIS MASTER'S STUDY PROGRAMME

Master's thesis

Grožio konkurso būdas modeliuoti atotrūkį nuo pusiausvyros baltijos šalyse

Beauty Contest Approach Measuring Output Gaps in Baltic States

Monika Milaknytė

Supervisor Asist. Prof., Dr. Dmitrij Celov

Vilnius, 2021

TABLE OF CONTENTS

ABSTRACT

The thesis estimates the potential output and the output gap for the economy of the Baltic states using several different techniques. The research begins with the output gap estimation using two commonly adapted univariate statistical filters: Hodrick-Prescott filter and Christiano-Fitzgerald filter. The analysis then proceeds to a more complex setting, introducing the "Beauty contest" concept, focusing on the model specification rather than on a prior selection of the methodology (conventional model "horse-race"). This presents a methodological approach of univariate unobserved components model based on structural time series and Kalman filter moving on to a bivariate setting that uses additional macroeconomic variables helping to improve the estimation results. The macroeconomic variables, which could accommodate specific cycles (demand, supply, financial, external, fiscal), are selected for the bivariate model based on introduced "beauty contest" selection criteria. The comparison between all methods used indicates that the introduction of additional variables improves the feature of output gap estimates, resulting in smoother, more well-defined and visible cycles with distinctive extrema.

Keywords: Baltic states, beauty-contest, business cycle, output gap, potential output, unobserved components model, Hodrick-Prescott filter, Christiano-Fitzgerald filter, Kalman filter

SANTRAUKA

Magistro baigiamasis darbas vertina produkcijos potencialą ir atotrūkį nuo pusiausvyros Baltijos šalyse naudojant keletą įvairių metodų. Atotrūkis nuo pusiausvyros pirmiausia vertinamas naudojant du statistinius filtrus, dažnai sutinkamus atotrūkio įvertinime: Hodrick-Prescott filtrą ir Christiano-Fitzgerald filtrą. Baigiamojo darbo analizė remiasi „grožio konkurso“ sąvoka, daugiau dėmesio skiriančia modelio specifikacijai nei išankstiniam metodikos pasirinkimui. Darbas pristato vienmatį nestebimų komponentų modelį, pagrįstą struktūriniu laiko eilučių modeliu ir Kalman filtru. Siekiant pagerinti atotrūkio nuo pusiausvyros įverčius, vienmatis modelis modifikuojamas į dvimatį įvedant papildomus makroekonominius veiksnius. Šie veiksniai atrenkami naudojant pasiūlytus atrankos kriterijus. Visų analizėje taikytų metodų palyginimas rodo, kad įvedus papildomus kintamuosius gaunami tikslesni atotrūkio nuo pusiausvyros įverčiai, apibrėžiantys sklandesnį verslo ciklą su geriau išreikštais ekstremumais.

Raktiniai žodžiai: atotrūkis nuo pusiausvyros, Baltijos šalys, grožio konkursas, produkcijos potencialas, nestebimų komponentų modelis, verslo ciklas, Hodrick-Prescott filtras, Christiano-Fitzgerald filtras, Kalman filtras

LIST OF ABREVIATIONS

AR	– Autoregressive
ARMA	– Autoregressive-Moving-Average
CF	– Christiano-Fitzgerald
CPI	– Consumer Price Index
EE	– Estonia
GDP	– Gross Domestic Product
HP	– Hodrick-Prescott
IMF	– International Monetary Fund
LT	– Lithuania
LV	– Latvia
MUC	– Multivariate Unobserved Components
NKPC	– New Keynesian Phillips Curve
OECD	– Organization for Economic Co-operation and Development
STS	– Structural Time Series
TFP	– Total Factor Productivity
TPC	– Traditional Phillips Curve
UC	– Unobserved Components
UCM	– Unobserved Components Model

INTRODUCTION

The concepts of the potential output and the output gap play a pivotal role in the domain of macroeconomic modelling of monetary and fiscal policies. Policymakers seek to recognize the specifics of the business cycle and its dynamics that help to determine the outcome of such policy decisions and ensure the stability of the financial system. However, as important the output gap is, it is also directly unobservable and thus comes with great uncertainty. For this reason, it is necessary to test various estimation techniques of the output gap and obtain results that meet mostly qualitative necessary and sufficient conditions. The latter are interpretable in the "beauty-contest" sense rather than seeking quantitative accuracy assessment in the "horse-race" sense conventionally applied in the similar econometric context for the actually observed data.

The literature on the potential output and the output gap estimation techniques is rather vast, ranging from simpler univariate methods, especially the commonly used various statistical filters, to methods relying on economic theory, as well as to multivariate techniques involving additional macroeconomic variables. The methods have various model specifications and seek to address country-specific issues. Despite the wide range of methods, the uncertainty surrounding the potential output and the output gap estimates still is to be the biggest issue. When the output gap estimates are faced with the statistical and economic optimality criteria, the results can be inconclusive, if the criteria are not well defined, then an effective selection algorithm is needed.

The aim of the thesis focuses on improving the existing output gap measurement methods and obtaining more precise estimates by introducing a novel bivariate unobserved components model together with the "Beauty contest" variable selection approach and put it to test for the data of the Baltic states. The "Beauty contest" term and selection criteria were introduced by Cuerpo, Cuevas and Quilis (2018) in their paper *Estimating Output Gap: a Beauty Contest Approach* [10]. Since the selection criteria in the article were developed for the Spanish data and might not fit the data fo the Baltic states, this thesis introduces an alternative set of criteria for more country-specific results.

In order to fulfil the aim of the thesis, certain objectives must be achieved. One must investigate existing literature about the output gap measurement methods for the broader understanding of the topic. Then the necessary data must be collected and prepared. For comparison of the estimates and the robustness checks, two conventional univariate output gap estimation methods are applied to the data as well – the Hodrick-Prescott filter and the Christiano-Fitzgerald filter. A set of alternative selection criteria are proposed and applied in the setting of the univariate unobserved components model and finally, the selected variables are modelled in the bivariate setting. The performance of all applied methods is compared, and the conclusions are drawn.

The thesis is structured as follows: Section 1 introduces the concept of the potential output and the output gap, its importance and usage, as well as existing commonly applied methods, that were not used in the thesis. The latter methods are overviewed because of their importance in terms of the historical development of the methodology and the general understanding of the trend-cycle decomposition methods spectrum. Section two introduces the methodology applied in the thesis, the proposed selection criteria and describes the data used for the analysis. Section 3 discusses the obtained results which lead to the drawn conclusions.

1. CONCEPTUAL AND METHODOLOGICAL ISSUES

1.1 Concepts of potential output and the output gap

The output gap is defined as the measure of the difference between the actual output that economy is producing and potential output, which is the maximum level of goods and services that economy can turn out while maintaining stable inflation through given time horizon [23]. Potential output is also referred to as *production capacity* of the economy or as *natural gross domestic product* and is expressed as a percentage of gross domestic product (GDP). The origin of the concept of potential output comes from the traditional theory of business cycles where output was decomposed into a deterministic trend and a stationary cycle component. Over time, the development of theory and econometrics led to the refinement of decomposition, which resulted in distinguishing of permanent and transitory stochastic components where a permanent component has been called potential output [11].

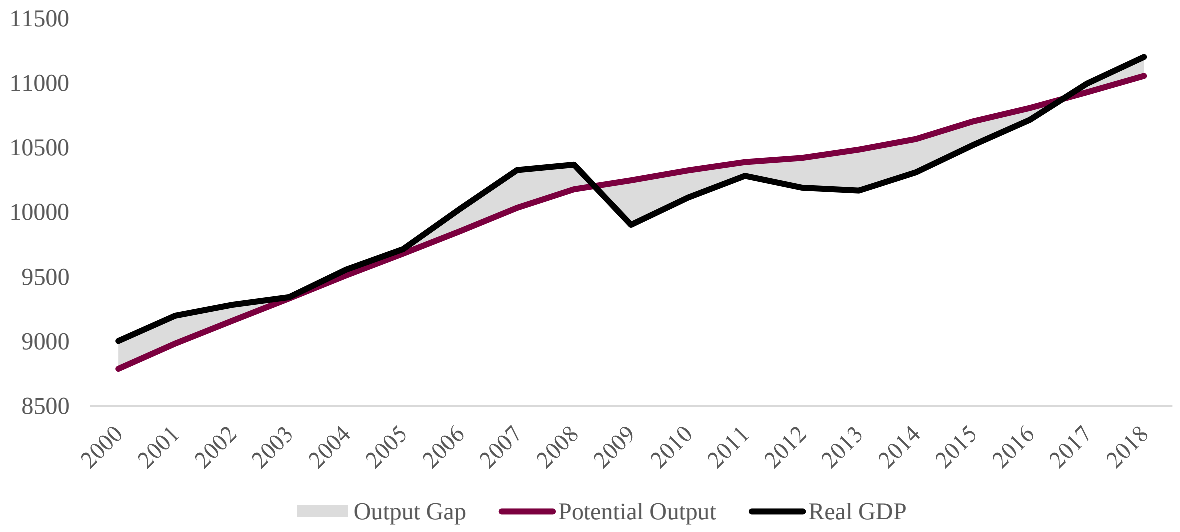


Figure 1. Euro Area Output Gap (2000 to 2018), billion € [24]

One of the first papers, discussing the output gap, was by Okun (1962), which later led to so-called “Okun’s law” linking level of output to level of unemployment [26]. Visualization of potential output and output gap of the Euro Area as a whole (in billion Euros) from 2000 up to 2018 can be seen in Figure 1, where one can observe negative output gap from 2009 to 2016 and slightly positive output gap from 2017.

Positive output gap occurs when actual output exceeds the potential output. This happens when actual GDP is above the long-run trend rate, when the economy is functioning at full capacity but an extra push in money supply and rise in aggregate demand can be observed (e.g., during an economic expansion). This demand can be satisfied in the short-term by firms increasing the wages and encouraging over-time work – one observes resources exceeding usual capacity. Such short-term solutions are not sustainable and usually lead to inflationary pressures.

Negative output gap (sometimes called deflationary or recessionary gap) occurs when actual output is below the potential output. In such a case, resources are under-utilized and the economy is producing less than its potential. This leads to increased unemployment rates, low economic growth and relative fall in output growth. Such an output gap might anticipate an upcoming recession (fall in GDP), yet not all growth rate decelerations reach negative growth region and growth cycle recessions will typically cause low inflation.

Despite being widely used for formulating policy recommendations, potential output, and therefore the output gap, is not directly observable and thus has an element of uncertainty. Critique of output gap measures has been extensively discussed in the literature [15]. For instance, Orphanides and Van Norden (2002) document how real-time estimates of the output gap for the US economy are highly unreliable [29] and Ho and Mauro (2014) state that long-term growth forecasts suffer from an "optimism bias", particularly for countries whose recent growth has been below trend [18]. Similar documentation can be found regarding other countries as well such as Norway [4], Canada [6], the United Kingdom [25], several OECD countries [31] and other [22]. It is also documented that incorrect output gap measurement has resulted in policies pursued by the Federal Reserve that led to the Great Inflation in the 1970s [28]. Output gap measurement mistakes also led to interest rate settings, which deviated around five hundred basis points from what would have been consistent with the actual output gap during so-called "Lawson boom" (unsustainable period of the fast growth of real GDP and rise in prices as a result of low-interest rates in the late 1980s) in the UK [25]. One of the recent campaigns launched by Brooks and Basile (2019) questions output gap measurement in the euro-area where several countries have roughly equal measures of output gaps despite different growth in per-capita GDP [5]. Despite the criticism, output gap estimates have evolved and improved over time. Modern computing and technologies have allowed analysts to incorporate a greater amount of information reducing measurement errors. Introduction of various business surveys in OECD countries reporting on the spare capacity of the economy is likely to improve estimates of the output gap as well [8].

Various approaches to measuring potential output are used in different countries and organizations [14]. For example, the one used by European Commission, and agreed with the Member States, is a production function method, where potential output depends on a combination of production factors such as labour and capital and total factor productivity (TFP), expressed at their trend level. The trend components of production factors are obtained using statistical filters (e.g., univariate Hodrick-Prescott filter or bivariate Kalman filter) [17]. Production function method is also used by the Organization for Economic Co-operation and Development (OECD), although it differs in the concept of structural unemployment: in the case of OECD, it corresponds to the equilibrium unemployment rate consistent with stable inflation (NAIRU) and is equal to the official target of the monetary authorities, thus incorporating inflation expectations [27]. The International Monetary Fund (IMF) uses different estimation methods depending on the country, using both multivariate filters and the production function [1].

1.2 Estimation methods

It is important to know the variety of approaches in the output gap assessment list used by analysts in order to understand the strengths and weaknesses of the methods and try suggesting useful improvements in the output gap measurement field. The methods applied in this thesis can be found in Chapter 2 while this subsection presents an overview of the methods commonly found in the literature. Several different approaches have been proposed for measuring potential output and the output gap. They range from simpler univariate ones, with particularly widely used various statistical filters, to methods relying on economic theory, as well as to more complex techniques requiring additional variables or even system-based approaches.

1.2.1 Univariate techniques

1.2.1.1 Linear trend

The simplest method of estimation is assuming that the trend component of output is a linear function of time. In this case, linear regression is performed on a constant and time trend of the log of real GDP (Y_t).

$$\ln Y_t = \alpha + \beta t + \varepsilon_{t.} \tag{1}$$

In this case t denotes time and potential output is expressed by the trend component ($\alpha + \beta t$). The method follows an assumption that real GDP can be decomposed into a deterministic trend component and a cyclical component modelled as a residual term ε_t. The main advantage of this method is its simplicity. However, the method does not allow any supply shocks to the system and implies a constant potential output growth rate. Moreover, the resulting gap might sometimes be non-stationary since the stochastic trend is not fully eliminated. Because of such reasons the method can bias the output gap by partially allocating trend components into the cyclical component.

1.2.1.2 Split time trend

This method is different from a linear trend in the sense that the trend output is calculated during each economic cycle - the period between peaks of economic growth:

$$\ln Y_t = \alpha + \sum_{j=1}^{n} \beta_i t_i + \varepsilon_t. \tag{2}$$

This model change allows estimated trend growth to change between cycles but not within each cycle. The method is also quite simple, although the difficulty may arise in determining the peaks [9].

1.2.1.3 Beveridge Nelson decomposition

In this model, trend and cycle are extracted from time series using identifying assumptions: one assumption is that the trend is modelled as a random walk while another is that shocks of trend-cycle are perfectly negatively correlated. The output gap is computed by transforming the real GDP series in stationary series and by then estimating an ARMA model which is used for series forecasting over a horizon *s*. The output gap is defined by:

$$c_t = E_t(\Delta y_{t+s} + \Delta y_{t+s-1} + \ldots + \Delta y_{t+1}) - s\hat{\alpha}, \tag{3}$$

where $\hat{\alpha}$ is the constant of the estimated ARMA model.

This filter does not have an end-sample problem common to some statistical filters like Hodrick-Prescott filter since it is a backward filter. However, it is time-consuming to compute and filter may generate noisy cycles. Additionally, one must choose between different ARMA models that may give quite different results and misrepresentation of $I(2)$ process as an $I(1)$ process may generate excess volatility in trend.

1.2.2 Multivariate techniques

1.2.2.1 Production function

The production function approach is based on neoclassical Solow-Swan economic growth theory. This method usually postulates a Cobb-Douglas technology where GDP is represented by a combination of production factor inputs – labour (L) and the capital stock (K). Assuming that technical progress is Harrod, Hicks and Solow neutral at the same time, the production function can be written as:

$$Y_t = (TFP_t\ L_t)^{\alpha} K_t^{1-\alpha} \tag{4}$$

where Y_t is the actual output, TFP_t – total factor productivity and α is the labour share. TFP is not observable and is usually computed as the Solow residual thus in log form has the following form:

$$tfp_t = 1/\alpha(y_t - (1-\alpha)k_t) - n_t. \tag{5}$$

Finally, potential output is calculated by substituting trend variables in the production function as well as with actual capital:

$$Y_t{}^* = (TFP_t^*\ L_t^*)^{\alpha} K_t^{1-\alpha} \tag{6}$$

where L_t^* is defined as $L_t^* = hrs_t^*, pop_t, pr_t^*, (1-u_t^*)$, $hrs*$ is an index of trend hours, *pop* is the population of working age, $pr*$ is the trend participation rate and $u*$ is the trend unemployment rate. The capital stock series is not detrended since the maximum potential contribution of capital is given by the full utilization of the existing capital in the economy. The downsides of this model are the choice of the model specification, poor quality of capital stock data and different assumptions on the trend components might lead to quite different estimates of the level of potential output [7].

1.2.2.2 Okun's law

American economist Arthur Okun in his 1962 paper has proposed a negative relation between the unemployment rate and changes in the output growth which became known as "Okun's law". The "levels" form of relationship can be written as:

$$U_t - U^* = \theta(Y_t - Y^*) \tag{7}$$

where *U* is the unemployment rate, *Y* is the logarithm of the actual output, *U** is "full employment" and *Y** is potential output. "Full employment" is an economic situation in which all available labour resources are used in the most efficient way possible. The difficulty arises with measuring the *U** and *Y** since they are unobservable, however it can be measure with methods like Hodrick-Prescott filter [21]. The downsides of the Okun's law are that unemployment rate is just a proxy variable for all the ways in which output is affected by idle resources. Furthermore, the unemployment rate is only one out of few factors determining the total amount of labour used as an input, others being a fraction of population present in the labour force and the number of hours worked by employed workers [2]. It has been suggested that Okun's law is unstable in many countries and that the relationship broke down during the economic recession of 2008-2009, where the correlation between the changes in output and unemployment was rather small [3].

1.2.2.3 Phillips curve

The role of the output gap affecting wage inflation was pioneered by economist Alban William Phillips (1958). In the short run, the Phillips curve indicates a positive relationship between the change in the price level and deviations of output relative to the potential for a given expected inflation rate. Since the potential output and output gap are not observable, it is appropriate to use multivariate unobserved components (MUC) models linked with the concept of the Phillips curve. Kuttner (1994) uses MUC model with Phillips curve in which the current change of inflation is related to the lagged output gap and a vector of additional variables to capture the effects of temporary relative shock on inflation [20]. The traditional Phillips curve (TPC) models of price adjustment state that the level of output relative to potential (C_t^{TPC}) is systematically related to inflation (π_t) and a set of exogenous variables, such as exchange rate or nominal oil prices:

$$\Delta\pi_t = \mu + \beta_0 C_{t-1}^{TPC}(y_t) + \beta_1 z_t + \varepsilon_t \qquad (8)$$

where z_t is exogenous variables, β_0 is the slope of the Phillips curve, β_1 – the elasticities of inflation concerning exogenous variables. Kuttner (1994) assumes that the output gap is an AR(2) process and potential output follows a random walk drift. In TPC formulation, inflation expectations are fully backwards-looking which means that expected inflation depends simply on lagged inflation. In a different approach called New Keynesian Phillips Curve (NKPC) model, forward-looking firms that maximize profits set prices based on expected marginal costs, so that current inflation depends on expected future inflation and the output gap. Doménech and Gómez (2006) use NKPC in their multivariate model:

$$\pi_t = \mu + \alpha E_t(\pi_{t+1}) + (1-\alpha)\pi_{t-1} + \beta_0 C_{t-1}^{NKPC}(y_t) + \beta_1 z_t + \varepsilon_t \qquad (9)$$

where the expected inflation is generated endogenously within the model [12]. The model includes the Phillips curve as well as Okun's Law and investment equation. It also follows the same assumption as TPC that the output gap is an AR(2) process and potential output follows a random walk with drift [2].

The overview of the methods indicates that the output gap estimation can be performed in many different ways, choosing either univariate or multivariate approach, using different specifications, incorporating economic theory and various supplementary variables. The methods are important since they are used in various institutions and countries and their estimates often determine the choice of certain policies. However, every mentioned method has some drawbacks and based on these drawbacks and model specifications were not chosen for the analysis, as they either do not fit the data or objectives of the thesis.

2. METHODOLOGY AND DATA

The methodology used for the estimation of the potential output and the output gap is presented in this section. The main goal is to introduce a multivariate unobserved components (MUC) approach based on the Structural Time Series (STS) representation of the time series by performing so-called "Beauty contest" variable selection using suggested criteria in seeking to improve the output gap estimates. Before proceeding to the multivariate setting, the data is analysed using univariate unobserved components (UC) approach and two statistical filters: Hodrick-Prescott (HP) filter and Christiano-Fitzgerald (CF) filter. The performance of the methods is compared, and conclusions are drawn. Estimation is performed for three Baltic countries – Lithuania, Latvia and Estonia. The data used for the analysis are seasonally and calendar adjusted quarterly and involves real GDP values and nine other macroeconomic covariate variables that would supplement the univariate UC turning it into a multivariate approach.

2.1 Hodrick-Prescott filter

The Hodrick-Prescott filter is one of the most popular choices when measuring the output gap and is essentially a simple smoothing procedure. Its popularity is high thanks to its flexibility in tracking the characteristics of the fluctuations in the trend output. Trend output derived using the filter is obtained by minimizing the gap between actual output, trend output and the rate of change in trend output [7]:

$$\min \sum_{t=1}^{T} (\ln Y_t - \ln Y_t^*)^2 + \lambda \sum_{t=1}^{T-1} [(\ln Y_{t+1}^* - \ln Y_t^*) - (\ln Y_t^* - \ln Y_{t-1}^*)]^2 , \lambda = \frac{\sigma_1^2}{\sigma_2^2} > 0, \quad (10)$$

where Y_t is the actual output, Y_t^* is the trend output, σ_1^2 is the variance of the output gap, σ_2^2 is the variance of the trend growth dynamics and λ is a Lagrange multiplier which is the smoothing (penalty) parameter determining the degree of smoothness of the trend. A low value of λ produces a trend which follows actual output closely, when values around between 1 and 2 could be used to remove the noise component of the series with frequencies below 1 year, and high value of λ reduces the sensitivity of the trend. Hodrick and Prescott (1993) provided arguments supported by stylized facts to set λ for the quarterly US data to 1,600. The authors suggest this value reasoning that *"[...] a 5 percent cyclical component is moderately large, as is a one-eighth of 1 percent change in the growth rate in a quarter."*. If the cycle components and the acceleration in trend components are independent normally distributed variables with zero mean, then the solution to (10) will be an optimal filter when $\sqrt{\lambda} = \sigma_1/\sigma_2 = 5/(1/8) = 40$ (λ=1600) [19]. However, this λ estimate drawn for US data may not guarantee accurate results when applied to other countries, for instance, when the point average length of the cycle differs from 8 years as is often found for the European economies. Therefore, the optimization of λ for specificity of each country would result in more precise values. The optimization algorithm can be found in Appendix G. Another drawback is that the filter has an unusual behaviour of cyclical components near the end of the sample. End-of-sample biases are particularly severe when analysing most recent observations in the sample to make projections for the immediate future and drive conclusions for policy implementations.

2.2 The Christiano-Fitzgerald filter

The Christiano-Fitzgerald filter is a band-pass filter formulated in the frequency domain. The data is filtered according to its frequency and it decomposes the time series into cycle eliminating both the trend and noise components [23]. The finite filter is given by:

$$c_t^* = b_0 y_t + \sum_{j=1}^{T-t-1} b_j y_{t+j} + \tilde{b}_{T-t} y_T + \sum_{j=1}^{t-2} b_j y_{t-j} + \tilde{b}_{t-t} y_1 \qquad (11)$$

where c_t^* is the cyclical component and $b_0, b_1, ..., b_j$ the weights from ideal band pass-filter:

$$\tilde{b}_{T-t} = -\frac{1}{2} b_0 - \sum_{j=1}^{T-t-1} b_j \qquad (12) \qquad \tilde{b}_{t-t} = -\frac{1}{2} b_0 - \sum_{j=1}^{t-2} b_j. \qquad (13)$$

There exists an ideal band pass filter of infinite length but since the time series are finite, the ideal filter is not precise and finite CF filter is used. Finite CF filter minimizes the mean squared error between series filtered by an ideal band pass filter and ones filtered by the finite filter. CF filter puts different weights to each observation thus is asymmetric and follows the assumption that the raw data is a random walk process [13]. Similarly to HP filter, CF filter faces an end-of-sample problem – the absence of future data makes it vulnerable to revision, although smaller one than for HP filter, though HP filter performs better at detecting signals of turning points [23].

2.3 The "Beauty contest" approach

2.3.1 The univariate unobserved components model

The univariate Unobserved Components (UC) model approach assumes that the observed time series data consist of the trend, the cycle and other components which cannot be observed directly. At first, GDP is decomposed as a sum of permanent component (p_t) and cyclical component (c_t) which are uncorrelated with each other, and an irregular component (ε_t):

$$y_t = p_t + c_t + \varepsilon_t. \tag{14}$$

In this model, the permanent component denotes the potential output estimate and the transitory component represent the output gap estimate. The permanent local trend specification:

$$p_t = g_{t-1} + p_{t-1} + \eta_t \tag{15}$$

$$g_t = g_{t-1} + \zeta_t \tag{16}$$

where η_t and ζ_t is an orthogonal white noise that admits particular cases dependent on restrictions of noise parameters η_t and ζ_t. The cyclical component is specified as an AR(2) to introduce persistence:

$$(1 - 2\rho\, cos(w_c)L + \rho^2 L^2)^c c_t = (1 - \rho\, cos(w_c)L)^c w_t, \tag{17}$$

where $\rho \in (0, 1)$ determines a damping factor of the cycle; w_c is a frequency in radians $\omega_c = 2\pi/\lambda_c$ corresponding to a cycle of length λ_c and w_t is white noise [7]. Combining the four equations above, the reduced-form MA model for y_t is given by:

$$y_t = p_t + c_t = \frac{1}{(1-B)^2}\eta_t + \frac{1}{(1-B)}\zeta_t + \frac{1}{(1-\rho\cos(w_c)B - \rho\cos(w_c)B^2)} w_t. \tag{18}$$

The three shocks that drive the system are assumed to be orthogonal Gaussian white noise innovations:

$$\begin{bmatrix}\eta_t \\ \zeta_t \\ w_t\end{bmatrix} \sim iid\ N\left(\begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix} v_\eta & 0 & 0 \\ 0 & v_\zeta & 0 \\ 0 & 0 & v_w\end{bmatrix}\right). \tag{19}$$

The structural model can be rewritten in a state-space format where corresponding transition and measurement equations are written as:

$$\underbrace{\begin{bmatrix} p_t \\ g_t \\ c_t \\ c_t^* \end{bmatrix}}_{S_t} = \underbrace{\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \rho\cos(w_c) & \rho\cos(w_c) \\ 0 & 0 & -\rho\sin(w_c) & \rho\cos(w_c) \end{bmatrix}}_{F} \underbrace{\begin{bmatrix} p_{t-1} \\ g_{t-1} \\ c_{t-1} \\ c_{t-1}^* \end{bmatrix}}_{S_{t-1}} + \underbrace{\begin{bmatrix} \eta_t \\ \zeta_t \\ w_t \\ 0 \end{bmatrix}}_{\zeta_t} \qquad (20)$$

$$y_t = \underbrace{[1 \quad 0 \quad 1 \quad 0]}_{H} \underbrace{\begin{bmatrix} p_t \\ g_t \\ c_t \\ c_t^* \end{bmatrix}}_{S_t} + \varepsilon_t. \qquad (21)$$

Unknown parameters are estimated by maximum likelihood. After obtaining the estimates, Kalman filter (one-sided) and Kalman smother (two-sided) are applied to obtain ex-ante and ex-post estimates of the corresponding cycle and trend. The parameters of the model can be placed in a single vector θ which is unknown and is estimated from the sample using the state-space form and Kalman filter which provide a suitable way to update the system states. When parameters of the vector θ have been estimated, the Kalman filter is applied to derive new initial conditions utilizing backcasting – forecasting observations prior the first observation, which is done by projecting forward the model using reversed time series. The output gap estimation using the Kalman filter is defined by estimation steps [10]:

1. Set initial parameters: θ_0.
2. Set initial conditions: S_0.
3. Maximum likelihood estimation of θ.
4. Setting new initial conditions $S_{0,1}$.
5. One-sided (concurrent) estimates of the state vector (Kalman filter).
6. Two-sided (historical) estimates of the state vector (Kalman smoother).

The unobserved components model has various specifications. In the article of Harvey and Jaeger (1993), the authors consider two models: local linear trend (the "unrestricted") model with a stochastic damped cycle (14-16), and a smooth trend (the "restricted") model, where $\sigma_\eta^2 = 0$, which transforms equation (6) to $p_t = g_{t-1} + p_{t-1}$ making the trend deterministic. A permanent (trend) component with this feature tends to be relatively smooth [16] and better fits stylised facts on macroeconomic data trends being represented by random walks with constant drifts.

2.3.2 Multivariate unobserved components model

The main methodology supporting this thesis the one proposed by Cuerpo, Cuevas and Quilis (2018) in their paper *Estimating Output Gap: a Beauty Contest Approach*. This paper introduces a novel approach of output gap measurement, where it is focused on the specification of the model instead of on prior selection of the methodology itself. The paper starts with the simple univariate model (described in the section presenting the univariate UC model of this thesis) and moves on to the multivariate setting. The multivariate approach uses the concept of "beauty contest", where candidate supplementing variables for the multivariate model are chosen using selection criteria. These criteria are applied while modelling actual GDP with potential variables containing relevant information about the business cycle in a bivariate setting. Variables fitting the selection criteria are used in the multivariate unobserved components model. Selection criteria are split into two categories: *necessary* (N) conditions reflecting statistical-based criteria and *sufficient* (S) criteria which are policy-related ones. The criteria can be found in Appendix A.

The criteria suggested in the article was applied to the Spanish data and might not fit the data of the Baltic states analysed in this thesis. An alternative set of "Beauty contest" criteria is proposed. The proposed criteria can be seen in Figure 2. The first criterion allows eliminating variables whose period cycle values (2π divided by estimated frequency cycle) after the estimation gave predefined lower or upper cycle boundary values which indicates that the cycle was not extracted and applied model did not fit the data. The second criterion eliminates variables by comparing visual estimated cycle representation of the additional variable with cycle representation of observed output and removing those that exhibit different cyclical path. The last step in variable selection is to estimate the correlation coefficient between the observed output and variables selected after the first two criteria. The correlation coefficient with values between 0.5-1 and (-1)-(-0.5) is selected for the bivariate setting.

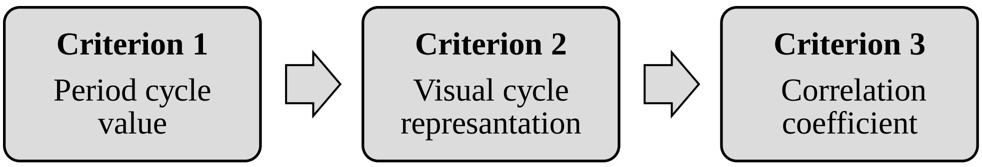


Figure 2. Suggested "Beauty contest" selection criteria for multivariate UC model

After applying the mentioned criteria to variables and selecting the ones that are a good fit, multivariate model, which extends its univariate counterpart by including these variables, is built. Addition of these variables allows the introduction of relevant macroeconomic stylized facts (such as Okun's Law, the Phillips curve, etc.) and provides additional information for output gap estimation.

The trend of additional variables can be I(1) or I(2). A simplified version representing two additional variables, one with an I(1) trend and the other I(2) trend respectively can be seen bellow.

$$
\begin{aligned}
y_{1,t} &= p_{1,t} + c_{1,t} + \varepsilon_{1,t} \\
p_{1,t} &= p_{1,t-1} + \eta_{1,t} \\
c_{1,t} &= \alpha_1 c_t + w_{1,t} \\
\begin{bmatrix} \eta_{1,t} \\ w_{1,t} \end{bmatrix} &\sim iid\ N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} v_{\eta,1} & 0 \\ 0 & v_{w,1} \end{bmatrix}\right)
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
y_{2,t} &= p_{2,t} + c_{2,t} + \varepsilon_{2,t} \\
p_{2,t} &= p_{2,t-1} + g_{2,t-1} + \eta_{2,t} \\
g_{2,t} &= g_{2,t} + \zeta_{2,t} \\
c_{2,t} &= \alpha_2 c_t + w_{2,t} \\
\begin{bmatrix} \eta_{2,t} \\ \zeta_{2,t} \\ w_{2,t} \end{bmatrix} &\sim iid\ N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} v_{\eta,2} & 0 & 0 \\ 0 & v_{\zeta,2} & 0 \\ 0 & 0 & w_{w,2} \end{bmatrix}\right).
\end{aligned}
\tag{23}
$$

The transition equation for the extended model and its corresponding measurement equation are written as:

$$
\underbrace{\begin{bmatrix} p_t \\ g_t \\ c_t \\ c_t^* \\ p_{1,t} \\ p_{2,t} \\ g_{2,t} \end{bmatrix}}_{S_t} = \underbrace{\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho\cos(w_c) & \rho\cos(w_c) & 0 & 0 & 0 \\ 0 & 0 & -\rho\sin(w_c) & \cos(w_c) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{F} \underbrace{\begin{bmatrix} p_{t-1} \\ g_{t-1} \\ c_{t-1} \\ c_t^* \\ p_{1,t-1} \\ p_{2,t-1} \\ g_{2,t-1} \end{bmatrix}}_{S_{t-1}} + \underbrace{\begin{bmatrix} \eta_t \\ \zeta_t \\ w_t \\ 0 \\ \eta_{1,t} \\ \eta_{2,t} \\ \zeta_{2,t} \end{bmatrix}}_{\zeta_t}
\tag{24}
$$

$$
\underbrace{\begin{bmatrix} y_t \\ y_{1,t} \\ y_{2,t} \end{bmatrix}}_{Y_t} = \underbrace{\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 & 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha_2 & 0 & 0 & 1 & 0 \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} p_t \\ g_t \\ c_t \\ c_{t^*} \\ p_{1,t} \\ p_{2,t} \\ g_{2,t} \end{bmatrix}}_{S_t} + \begin{bmatrix} 0 \\ w_{1,t} \\ w_{2,t} \end{bmatrix}.
\tag{25}
$$

The algorithm for parameter estimation is the same as in the univariate UC case [10].

2.4 The data

The thesis analyses the quarterly data of three Baltic states – Lithuania (LT), Latvia (LV) and Estonia (EE). Each country has the main variable – real GDP, and nine supplementary macroeconomic variables, which share relevant information about the business cycle, from fields such as domestic economy, external sector, prices, labour market and fiscal conditions variables. The data spans from the first quarter of 1995 to the third quarter of 2020 resulting in 103 observations for each variable. Some of the variables have a shorter time series with fewer observations. All data are corrected for seasonal and calendar effects which allows calculating the cyclical component of the series more accurately. The data were obtained from Eurostat, OECD and the World Bank. Selected variables of each country, their units, time range and the source are listed in Table 1. The variables were chosen based on the ones suggested by Cuerpo, Cuevas and Quilis (2018) and the literature cited in the paper, selecting those with frequency matching the GDP variable (quarterly) and of an appropriate length. Noticeably short time series were not included. All variables were transformed into a logarithmic scale for the analysis.

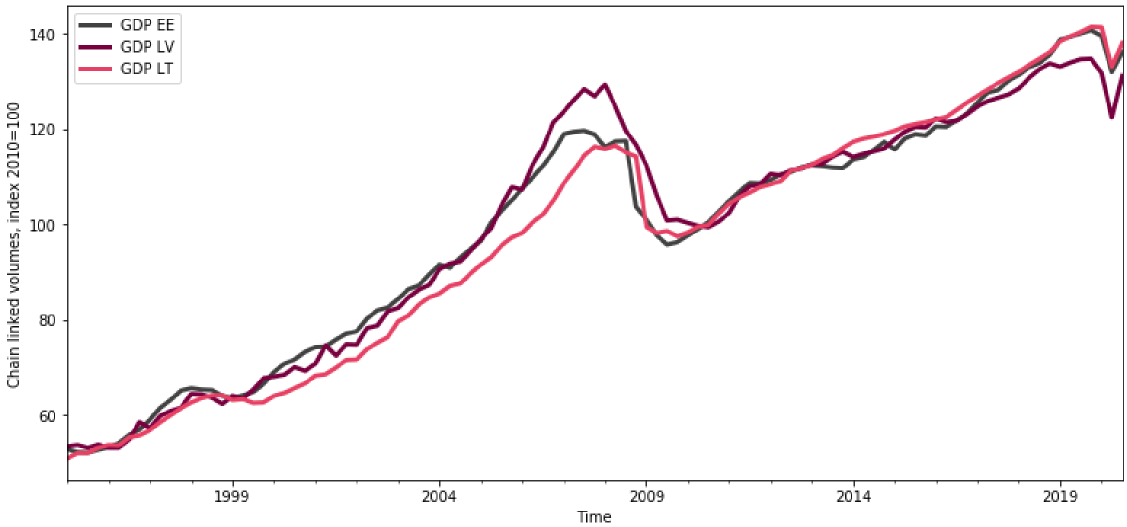


Figure 3. GDP of the Baltic States (1995 Q1 to 2020 Q3)

The main variable of the analysis – GDP – has exhibited an upward trend for all the Baltic countries. With a big fall during the crisis of 2008 and noticeable drop in the second quarter of 2020 concerning the effects of the COVID-19 pandemic, all countries follow a similar path, with Latvia having slightly greater fluctuations in values. The GDP values of all three countries are exhibited in Figure 3.

The current level of capacity utilization is a very useful variable since it shows the percentage of potential output that is being utilized and is likely a very perspective candidate for supplementing the GDP variable in the output gap estimation. Nominal effective exchange rate and current account balance are candidates which give information about the external sector and indicate the country‘s interaction with foreign economies. Consumer Price Index (CPI) is frequently used for identifying periods of inflation or deflation and thus is a standard inflationary pressure representing variable in the output gap estimation. The importance of unemployment rate variable for the output gap measurement is not deniable as it has already been discussed when it comes to Okun‘s law and together with compensation of employees are great indicators of the labour market and its changes. The last three variables (Gross Public Sector Debt, Net lending/borrowing and Taxes on Production and Imports) are fiscal indicators and are closely related with the output gap since governments can adjust fiscal policies to close the output gap. Thus all macroeconomic variables used in the analysis are rather important and relevant when it comes to improving the output gap estimates. Descriptive statistics of the data set can be found in Appendix B.

Table 1. The data set

Variable	Unit	Number of observations	Time range	Source
LT GPD at market prices	Chain linked volumes, index 2010=100	103	1995Q1-2020Q3	Eurostat
LV GDP at market prices				
EE GDP at market prices				
LT Current level of capacity utilization	%	103	1995Q1-2020Q3	Eurostat
LV Current level of capacity utilization				
EE Current level of capacity utilization				
LT Nominal effective exchange rate - 42 trading partners (industrial countries)	Index, 2010=100	103	1995Q1-2020Q3	Eurostat
LV Nominal effective exchange rate - 42 trading partners (industrial countries)				
EE Nominal effective exchange rate - 42 trading partners (industrial countries)				
LT Current account balance	% GDP	102	1995Q1-2020Q2	Eurostat
LV Current account balance		82	2000Q1-2020Q2	
EE Current account balance		102	1995Q1-2020Q2	
LT CPI	Index, 2015=100	103	1995Q1-2020Q3	OECD
LV CPI		103	1995Q1-2020Q3	
EE CPI		91	1998Q1-2020Q3	
LT Unemployment rate	%	90	1998Q2-2020Q3	OECD
LV Unemployment rate				
EE Unemployment rate				
LT Compensation of employees	Current prices, M€	103	1995Q1	Eurostat
LV Compensation of employees				
EE Compensation of employees				
LT Gross Public Sector Debt, General Gov.	% GDP	87	1998Q-2020Q2	The World Bank
LV Gross Public Sector Debt, General Gov.		82	2000Q1-2020Q2	
EE Gross Public Sector Debt, General Gov.		87	1998Q-2020Q2	
LT Net lending/borrowing (current and capital account)	% GDP	102	1995Q1-2020Q2	Eurostat
LV Net lending/borrowing (current and capital account)		82	2000Q1-2020Q2	
EE Net lending/borrowing (current and capital account)		102	1995Q1-2020Q2	
LT Taxes on Production and Imports	Current prices, M€	103	1995Q1	Eurostat
LV Taxes on Production and Imports				
EE Taxes on Production and Imports				

3. ESTIMATION RESULTS AND DISCUSSION

This section provides the results of methods used for the potential output and the output gap estimation. The estimated results of the potential output and the output gap are described for every model separately as well as compared across these methods.

3.1 Hodrick-Prescott filter

The potential output and the output gap estimates were obtained using HP filter for the GDP data of each country. As mentioned before, in order to obtain more country-specific results, the smoothing parameter λ, which is the only explicit choice of the model was optimized, obtaining specific value for each country instead of choosing $\lambda = 1600$ as a standard practice for quarterly data. Each country was modelled to filter out a cycle of around 10 (or fewer) years or around 40 quarters. This resulted in $\lambda = 3971$ value for Lithuania, $\lambda = 3929$ value for Latvia and $\lambda = 3850$ value for Estonia.

Figure 4 depicts the estimated trend component against the observed GDP and the cycle component of each country, where the trend corresponds to the estimated potential output and the cycle corresponds to the output gap. All countries follow a similar cyclical pattern, with Latvia having nosier cycle than Lithuania and Estonia. One can observe three more or less distinctive pitfalls below potential, the first one indicating the impact of the Russian financial crisis in 1998, the second one being the Great Recession of 2008 with the biggest widening of the output gap to the negative side, and the third one representing the impact of the currently ongoing COVID-19 pandemic. The end of the sample problem, specific to the HP cycle, is also visible in Figure 4, where the value of the output gap is largely driven by the actual observed output. Descriptive statistics of the estimated cycle for each country are listed in Table 2.

Table 2. Descriptive statistics of the output gap HP filter estimates

Country	Mean	Median	Min	Max	Std. Dev.
Lithuania	$4.28 \cdot 10^{-11}$	-0.04	-8.93	11.75	4.01
Latvia	$-1.18 \cdot 10^{-11}$	-0.58	-11.04	14.81	5.06
Estonia	$-9.74 \cdot 10^{-11}$	-0.61	-11.64	11.71	4.44

The maximum and minimum of the estimated output gap range between -8.93 % and 11.75 % for Lithuania, -11.04 % and 14.81 % for Latvia, -11.64 % and 11.71 % for Estonia, maximum values being just before the recession of 2008 and minimum values – during the global financial crisis. The mean values of the cycles are assessed as technical zeroes indicating that all three countries have been in both the recessionary and the expansionary cycle for around the same amount of time for the observed period.

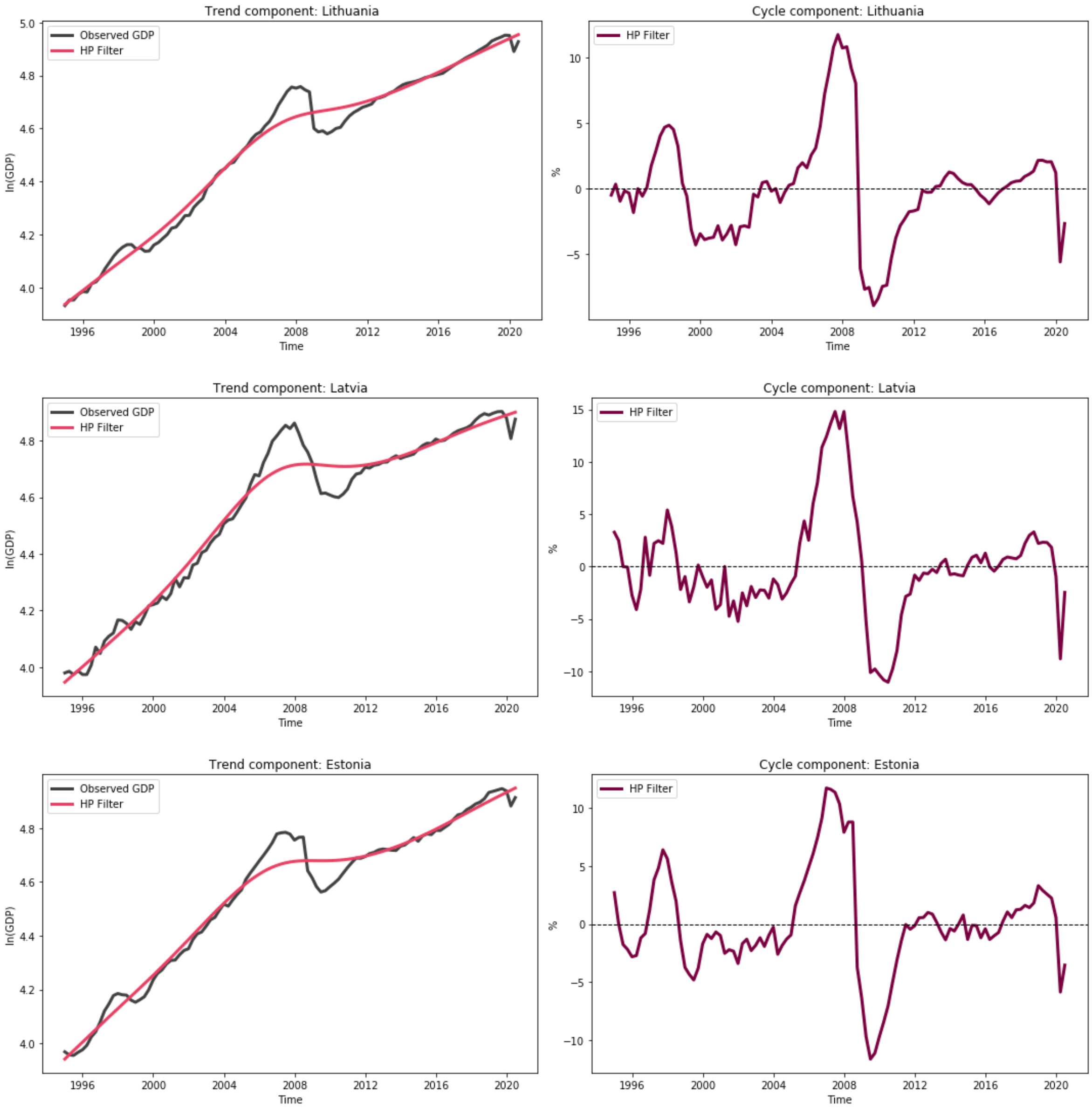


Figure 4. HP filter estimation results

3.2 Christiano-Fitzgerald filter

Another statistical filter used is the Christiano-Fitzgerald filter. The filter requires a choice of the cut off lengths of the cycle, where cycles between the minimum and the maximum number of oscillations are considered to be the cyclical component of the data. Usually when applying the CF filter, the default choice of the minimum bound is set to be 6 quarters or 1.5 years and the maximum one is 32 quarters or 8 years for quarterly data. The cut-off values chosen for this analysis were 6 and 40, increasing the upper bound to 10 years, similarly to the one used in HP filter analysis.

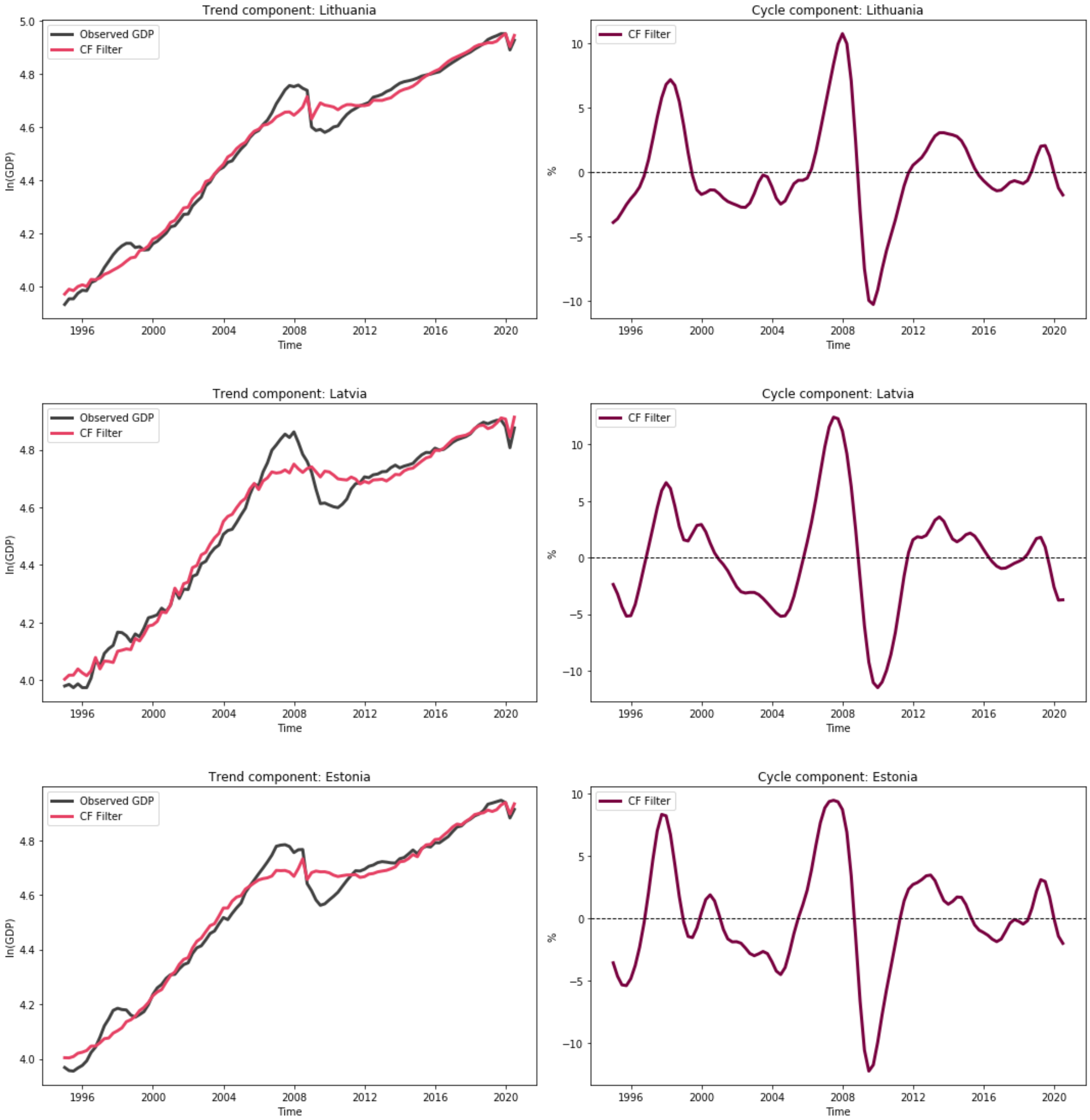


Figure 5. CF filter estimation results

The results estimated with the CF filter follow a similar pattern as the ones obtained with HP filter. The trend is more volatile resulting in a much smoother cycle than HP cycle. Descriptive statistics in Table 3 indicate, that the cycle values span larger to both negative and positive side compared to HP cycle and have a greater mean value.

Table 3. Descriptive statistics of the output gap CF filter estimates

Country	Mean	Median	Min	Max	Std. Dev.
Lithuania	0.07	-0.65	-10.26	10.74	3.83
Latvia	-0.02	0.14	-11.50	12.42	4.74
Estonia	0.02	-0.25	-12.25	9.45	4.36

3.3 Univariate unobserved components model and variable selection

The analysis using univariate unobserved components model was performed for not only the GDP data of each country but also with all the variables of the data set. Every variable was analysed in a univariate setting aiming to select appropriate candidates for the bivariate model – the process referred as the "Beauty contest", where the "prettiest" (most suitable) variable based on the selection criteria later accompanies the GDP data in a bivariate setting.

The first stage of the variable selection requires comparing the model estimates obtained with the UC model. The irregular term of the model is assumed to be white noise, and the cycle is assumed to be stochastic and damped. The period cycle bounds are set to be between 1.5 and 12 years or 6 and 48 quarters. The trend component has two types of model specifications for comparison – the "unrestricted" local linear trend model and "restricted" smooth trend model, where the variance of the trend component is assumed to be zero ($\sigma_\eta^2 = 0$). Comparison of the estimates will be conducted in the setting of the "restricted" smooth trend specification to obtain a smooth trend and cycle.

3.3.1 Variable selection for Lithuania's data

First, the estimation results of the UC model are described similarly to the two statistical filters. Figure 6 shows that the model estimates of the UC model follow a similar pattern but within a smaller deviation amplitude from the potential output. The first four observations of the UC model estimates are not shown due to approximate diffuse initialization. Minimum value is -6.64 %, maximum value is 7.18 % (mean: -0.04 %, median: -0.25 %, std. dev.: 2.60 %). The cycle has an almost distinctive oscillatory pattern with more defined peaks and troughs.

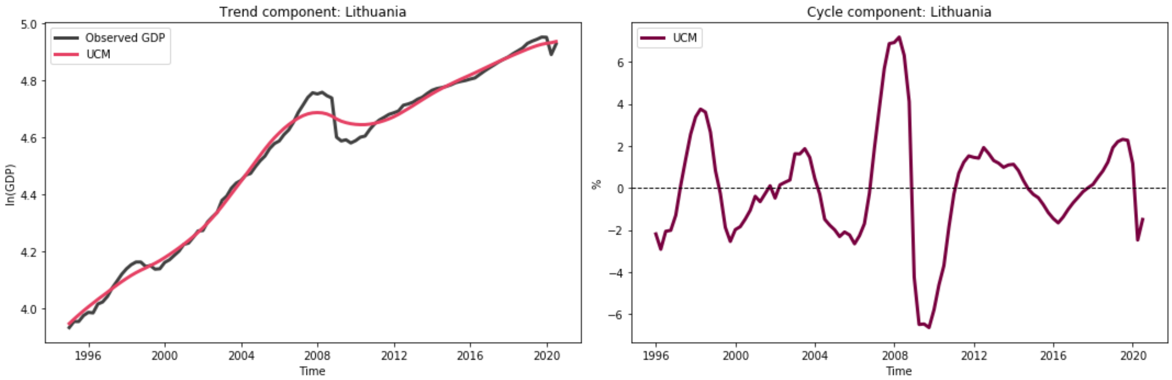


Figure 6. UC model estimation results for Lithuania's data

Table 4 contains the estimates of both model specifications and is composed similarly as Table I of Harvey and Jaeger. The first coefficient in the table is the variance of the level component, the second – the trend component, the third – the cycle component, $\rho \in (0, 1)$ is the damping factor, the fifth coefficient column indicates the period cycle (in quarters) which is calculated by dividing 2π with the estimated frequency cycle. The last coefficient is the variance of the irregular component. All values except the damping factor and the period cycle are scaled up by multiplying 1e7.

Table 4. Estimates of the UC models for Lithuania's data

Variable	Restrictions	σ_ζ^2	σ_η^2	σ_w^2	ρ	$2\pi/\lambda_c$	σ_ε^2
GDP	-	0	140.1	1163	0.93	18.56	506.4
	$\sigma_\eta^2 = 0$	137.6	-	1158	0.93	18.55	504
Current level of capacity utilization	-	0	115.1	1461	0.93	18.08	1047
	$\sigma_\eta^2 = 0$	115	-	1461	0.93	18.08	1047
Nominal effective exchange rate	-	5173	238.6	0	0.95	8.14	0
	$\sigma_\eta^2 = 0$	351.1	-	3439	0.76	12.62	0
Current account balance	-	532800	0	209500	0	6	2053
	$\sigma_\eta^2 = 0$	0	-	840200	0.7	48	30.5
CPI	-	99.67	88.26	497.9	0.64	20.57	0
	$\sigma_\eta^2 = 0$	97.1	-	558.4	0.65	20.16	0
Unemployment rate	-	4141	21110	6255	0.14	6	0.46
	$\sigma_\eta^2 = 0$	22990	-	3482	0.02	6	3586
Compensation of employees	-	0.01	181.2	4969	0.92	28.74	684.5
	$\sigma_\eta^2 = 0$	181.3	-	4969	0.92	28.73	684.6
Public Sector Debt	-	0	8033	0	0.74	6.03	45850
	$\sigma_\eta^2 = 0$	8042	-	0	0.74	6.03	45880
Net lending/borrowing	-	6109	0	348200	0.94	24.21	0
	$\sigma_\eta^2 = 0$	0	-	354500	0.94	24.48	0
Taxes on production and imports	-	2757	25.46	7529	0.9	22.89	0.03
	$\sigma_\eta^2 = 0$	37.48	-	9744	0.9	23.47	5.97

The estimation for the GDP variable resulted in a period cycle of 18.55 quarters or approximately 5 years (arguably short business cycle), while also taking into account the damping factor, that adds to the persistence of the business cycle. Irregular component suggests that there is some error in the effective trend and cycle decomposition of the data. When looking at the estimates of other variables, the first criterion of selection instantly eliminates those with highly different period cycle values than the ones of the GDP. In this way, four variables are eliminated: nominal effective exchange rate which resulted in the even shorter business cycle (12.62 quarters) and other three variables, where the model did not fit the data – current account balance, unemployment rate and public sector debt. These three variables adapted the predetermined period cycle bounds of either lower one of 6 quarters or upper one of 48 quarters and the cycle was not extracted.

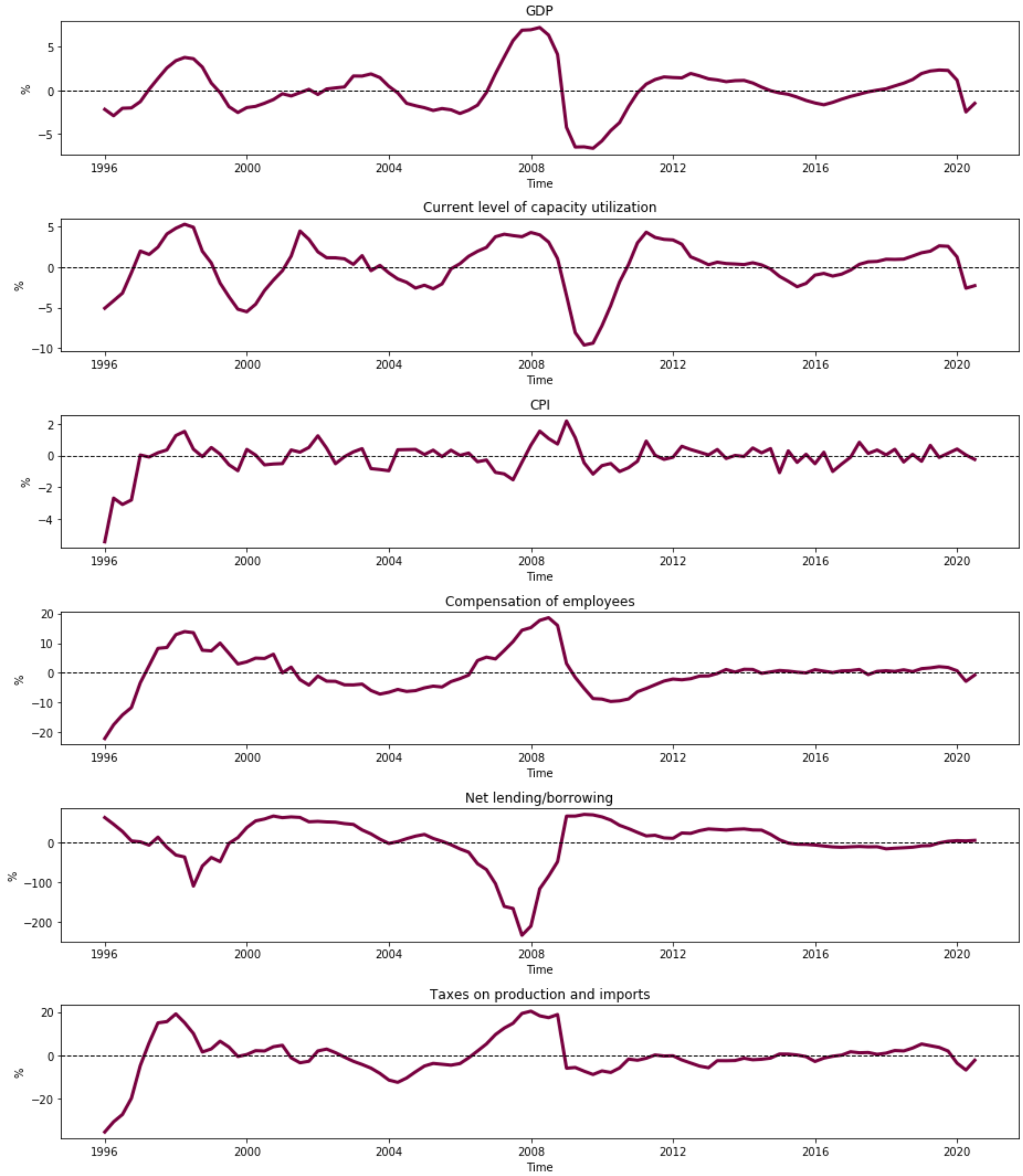


Figure 7. UC cycle estimates of the remaining variables (Lithuania's data)

After applying the first criterion, five potential candidate variables remain. The second criterion requires a visual comparison between the cyclical components, removing variables exhibiting a highly different pattern than the one of the GDP. Figure 7 shows the output gap estimate for each of the variable. One can observe that the CPI variable follows a different pattern and has a noisier cycle. Other variables cannot be rejected based only on their cyclical pattern and thus move on to the last criterion.

Table 5. Correlation coefficient values of Lithuania's data

	Current level of capacity utilization	Compensation of employees	Net lending/borrowing	Taxes on production and imports
GDP	0.80	0.55	-0.68	0.57

The last criterion requires examining the correlation coefficient between the GDP and the four remaining variables. For a variable to be chosen for the bivariate model, its correlation coefficient with the GDP variable has to be greater than ±0.5. Table 5 indicates that all four remaining variables pass this criterion, where current level of capacity utilization has the highest correlation coefficient value, compensation of employees and taxes on production and imports have a similar value, and net lending/borrowing is the single negatively correlated variable.

3.3.2 Variable selection for Latvia's data

When estimating Latvia's GDP data with the same model specification as for Lithuania's data, the results were not satisfactory: the period cycle estimate was equal to 6 which indicated that the cycle was not extracted. This resulted in one change of the specification – cycle is set to be stochastic instead of stochastic and damped, and the damping factor ρ is restricted to 1. Other variables have the same specifics as for Lithuania's model.

The estimated cycle for Latvia's data has a highly smooth pattern with almost sinusoidal path and very distinctive peaks and valleys. The deviations from the potential output are much smaller when comparing to the estimates of the statistical filters, minimum value being -5.59 % and maximum value of 5.77 % (mean: -0.05 %, median: -0.05 %, std. dev.: 2.63 %).

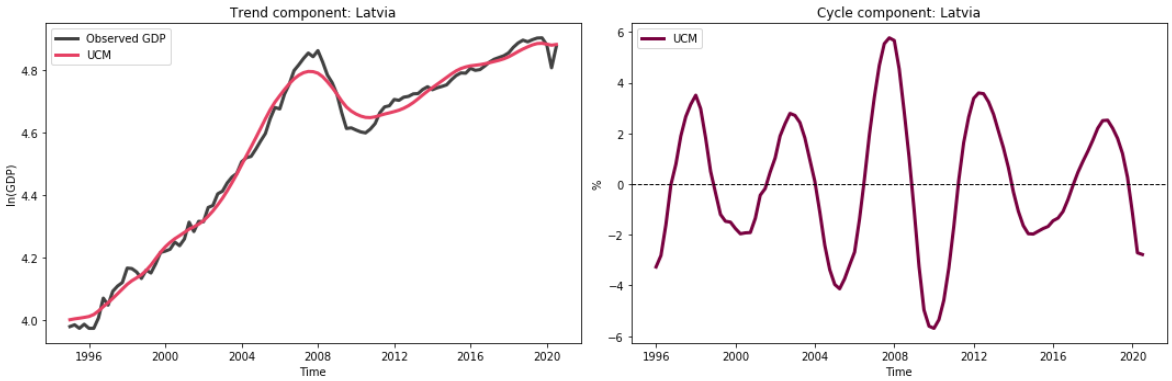


Figure 8. UC model estimation results for Latvia's data

Table 6 displays the estimated coefficient values of Latvia's data. Similarly to Lithuania's data, the estimation for the GDP variable resulted in a period cycle of 19.79 quarters or approximately

5 years. The period cycle value allows eliminating two variables: the nominal effective exchange rate and compensation of employees, which gave the lower bound value. The period cycle of current level of capacity utilization in the local linear trend model did not extract the cycle, giving the value of the upper bound, but since the smooth trend model is the one to consider in the variable selection, the variable proceeds to the next stage.

Table 6. Estimates of the UC models for Latvia's data

Variable	Restrictions	σ_ζ^2	σ_η^2	σ_w^2	ρ	$2\pi/\lambda_c$	σ_ε^2
GDP	-	0	354.4	371.7	1	19.79	1000
	$\sigma_\eta^2 = 0$	354.2	-	3174000	1	19.79	1000
Current level of capacity	-	15700	0	2441	0	48	162.9
utilization	$\sigma_\eta^2 = 0$	151.1	-	7988	0.85	19.3	4280
Nominal effective	-	443.7	785.9	1289	0.67	6.43	0
exchange rate	$\sigma_\eta^2 = 0$	839.6	-	1461	0.66	6.54	0
Current account balance	-	293900	0	158600	0.93	19.19	0.44
	$\sigma_\eta^2 = 0$	2697	-	350500	0.91	21.97	16460
CPI	-	103.4	106.5	618.8	0.84	21.09	0
	$\sigma_\eta^2 = 0$	105.7	-	697.7	0.83	21.39	0
Unemployment rate	-	0	3700	23630	0.96	22.21	3559
	$\sigma_\eta^2 = 0$	3701	-	23620	0.96	22.21	3563
Compensation of	-	0	4257	9.18	0.61	6	2254
employees	$\sigma_\eta^2 = 0$	4258	-	8.91	0.61	6	2254
Public Sector Debt	-	0	2825	10050	0.97	25.37	7752
	$\sigma_\eta^2 = 0$	2824	-	10050	0.97	25.37	7751
Net lending/borrowing	-	0	0	421200	0.91	28.28	92160
	$\sigma_\eta^2 = 0$	0	-	421200	0.91	28.28	92160
Taxes on production and	-	1823	38.13	8937	0.95	26.36	5891
imports	$\sigma_\eta^2 = 0$	67.75	-	9342	0.95	26.33	6404

Seven potential candidate variables remain after applying the first criterion. Visual comparison of estimated cycles in Appendix C allows eliminating the current account balance and net lending/borrowing variables since they exhibit a different cycle pattern. Remaining five variables require investigating the correlation coefficient for a finite approval as a variable for the bivariate model.

Table 7. Correlation coefficient values of Latvia's data

	Current level of capacity utilization	CPI	Unemployment rate	Public sector debt	Taxes on production and imports
GDP	0.68	0.30	-0.77	-0.54	0.60

Table 7 indicates that the CPI variable has a correlation coefficient in absolute value below 0.5 and thus is rejected as a candidate variable. The remaining four variables have a strong correlation with GDP, current level of capacity utilization having the highest positive correlation coefficient and unemployment rate having the strongest negative correlation.

3.3.3 Variable selection for Estonia's data

Estonia's model follows the same model specifics as for Lithuania's model for all the variables. The cycle estimated by UC model exhibits a similar pattern to the ones estimated by statistical filters, being closer to the HP estimates in terms of cycle variability. Similarly to other countries, Estonia's cycle has lower deviations from the potential output, minimum value being -9.30 %, maximum value is 6.80 % (mean: -0.24 %, median: -0.03 %, std. dev.: 3.44 %).

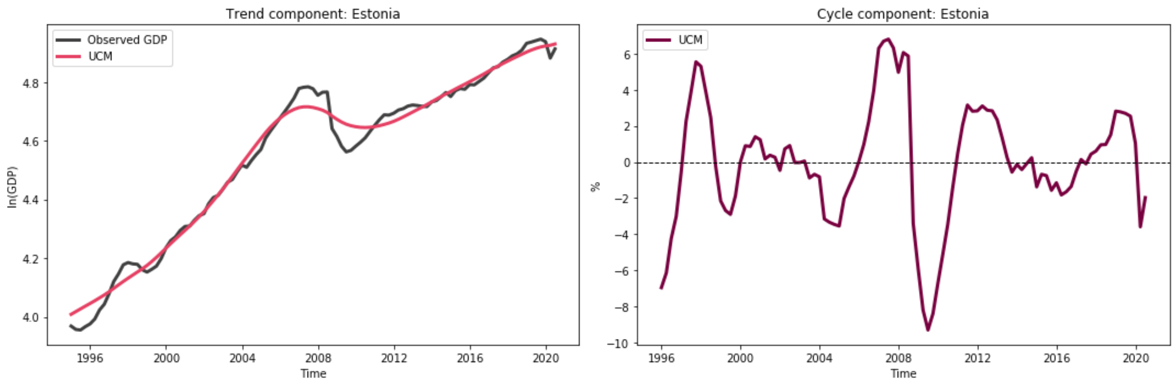


Figure 9. UC model estimation results for Estonia's data

Table 8 displays the estimated coefficient values of Estonia's data. Similarly to the other two countries data, the estimation for the GDP variable resulted in a period cycle of 19.59 quarters or approximately 5 years, while also taking into account the damping factor. The period cycle value allows eliminating two variables: net lending/borrowing and taxes on production and imports, which gave the upper bound value. Supplementary variables in general have shorter period cycles when compared to the ones estimated for Lithuania and Latvia.

Seven potential candidate variables remain after applying the first criterion. Visual comparison of estimated cycles in Appendix D allows eliminating the nominal effective exchange rate variable since it exhibits a different cycle pattern. Remaining six variables require investigating the correlation coefficient for a final approval as a variable for the bivariate model.

Table 8. Estimates of the UC models for Estonia's data

Variable	Restrictions	σ_ζ^2	σ_η^2	σ_w^2	ρ	$2\pi/\lambda_c$	σ_ε^2
GDP	-	0	147.1	1533	0.93	19.59	228.8
	$\sigma_\eta^2 = 0$	147.1	-	1533	0.93	19.59	228.7
Current level of capacity utilization	-	0	87.45	3694	0.92	17.39	1371
	$\sigma_\eta^2 = 0$	87.42	-	3694	0.92	17.39	1371
Nominal effective exchange rate	-	0	31.92	3192	0.86	15.13	0
	$\sigma_\eta^2 = 0$	31.92	-	3192	0.86	15.14	0
Current account balance	-	237300	0.01	93610	0.93	16.97	204100
	$\sigma_\eta^2 = 0$	5135	-	146000	0.93	17.16	263000
CPI	-	24.69	64.08	289.8	0.87	14.34	0
	$\sigma_\eta^2 = 0$	62.21	-	307.9	0.87	14.58	0
Unemployment rate	-	0	15060	14170	0.95	17.22	23840
	$\sigma_\eta^2 = 0$	15060	-	14170	0.95	17.22	23840
Compensation of employees	-	0	308.9	1843	0.95	21.32	233.3
	$\sigma_\eta^2 = 0$	308.7	-	1843	0.95	21.32	233.5
Public Sector Debt	-	0	0	78670	0.79	18.9	0
	$\sigma_\eta^2 = 0$	0	-	78670	0.79	18.9	0
Net lending/borrowing	-	586300	0	513700	0	48	49610
	$\sigma_\eta^2 = 0$	4856	-	646600	0.72	48	477800
Taxes on production and imports	-	13770	55.16	35070	0	48	21.06
	$\sigma_\eta^2 = 0$	1198	-	38840	0	48	243.3

Table 9 indicates that CPI and public sector debt variables have a low correlation coefficient and thus are rejected as candidate variables. The remaining four variables have a strong correlation with GDP, current level of capacity utilization having the highest positive correlation coefficient and current account balance having the strongest negative correlation. As in the case of Latvia, selection resulted in two positively and two negatively with GDP correlated variables.

Table 9. Correlation coefficient values of the Estonian data

	Current level of capacity utilization	Current account balance	CPI	Unemployment rate	Compensation of employees	Public sector debt
GDP	0.72	-0.71	0.36	-0.59	0.66	-0.40

3.3.4 Variable selection results in comparison

After applying the “Beauty contest” criteria, variables suiting all requirements were selected. The number of selected variables for each Baltic state is the same – four variables out of the starting nine. The results of selection can also indicate some insights about the economy of each country.

1. Current level of capacity utilization variable was selected for all three countries, which comes as no surprise: as mentioned before, it is an important business cycle indicator, as it relates directly to the stress on the current capacity to produce goods and services.
2. Unemployment rate variable was an expected candidate for all three countries (referring to Okun’s law) and in the case of Lithuania, the variable is simply not selected because of the poor cycle estimation in the first step of selection; model parameter adjustments would most likely result in the variable being selected.
3. Some of the fiscal variables (public sector debt, net lending/borrowing and taxes on production and imports) were chosen for cases of Lithuania and Latvia but not for Estonia, which highlights Estonia’s cautious, prudent and deficit-reducing fiscal policies that resulted in lowest deterioration of the fiscal position during 2008 crisis out of the three countries, whereas Lithuania and Latvia had one of the most procyclical fiscal policies.

In general, variables that were selected in Estonia’s case are usually the typical variables to be selected for the improvement of the output gap estimation and show that Estonia’s economic indicators are somehow closer to the ones expected for the countries around the Eurozone average. The summarized selection results for Baltic states are displayed in Table 10, where grey cells indicate the selected variable.

Table 10. “Beauty contest” variable selection results

	Lithuania	Latvia	Estonia
Current level of capacity utilization			
Nominal effective exchange rate			
Current account balance			
CPI			
Unemployment rate			
Compensation of employees			
Public sector debt			
Net lending/borrowing			
Taxes on production and imports			

3.4 Bivariate unobserved components model

The variables satisfying the selection criteria are modelled in a bivariate setting together with the GDP variable. This results in four bivariate unobserved components models for each country, twelve models in total. The GDP time series were adjusted in those cases when the supplementary variable had fewer observations: quarters of the GDP data were removed to match the time series of the covariate. The graphical visualization of trend and cycle estimates can be found in Appendix E.

In the case of Lithuania, bivariate model in the Figure E.1. between the GDP and current level of capacity utilization exhibits a similar pattern to the univariate output gap model but with even more distinct peaks. The trend has an interesting pattern at around 2008 where one can observe a very sudden fall in the potential. This structural break could indicate a very sudden change in the tax system referred to as the "night reform". Other three variable cycle estimates (Figures E.2., E.3 and E.4) have longer cycle periods and sharper peaks (especially visible for net lending/borrowing variable) and greater deviation from the potential. Figure 10 displays all four bivariate models and their averaged value. Main differences between estimates can be seen around the period of 2001-2004 and 2012-2014, where some variables exhibit a positive output gap while others are negative. The averaged cycle could serve as an alternative result, which includes the cyclical features of all four variables thus giving a broader understanding of changes and their impact in different aspects of the economy.

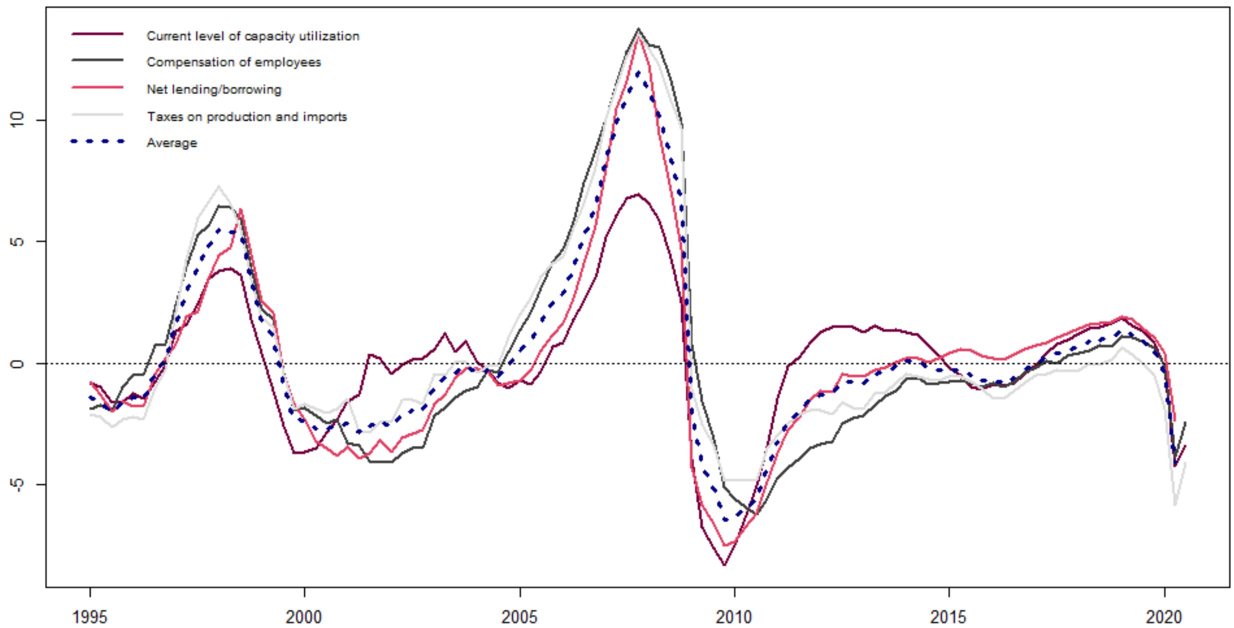


Figure 10. Bivariate UC model estimates and their average, Lithuania's data

In the case of Latvia, the bivariate model with the current level of capacity utilization in Figure D.5. give different results than the univariate estimate of the output gap, with observably longer cycles and fewer peaks. Models with the unemployment rate and gross public sector debt (Figures E.6. and E.7.) exhibit very long cycles, with only two big peaks. The model with taxes on production and imports (Figure E.8.) highlight the abnormal overheating period in 2007 and have a less distinctive pattern for other periods, having values close to the potential. One can observe the overall cyclical pattern of Latvia's estimates in Figure 11 depicting all four bivariate models and their average. Some variables have fewer observations (unemployment rate, public sector debt) thus the averaged cycle does not begin before 2000. The estimates at the beginning of the observed period have noisier pattern and only two definite peaks can be distinguished.

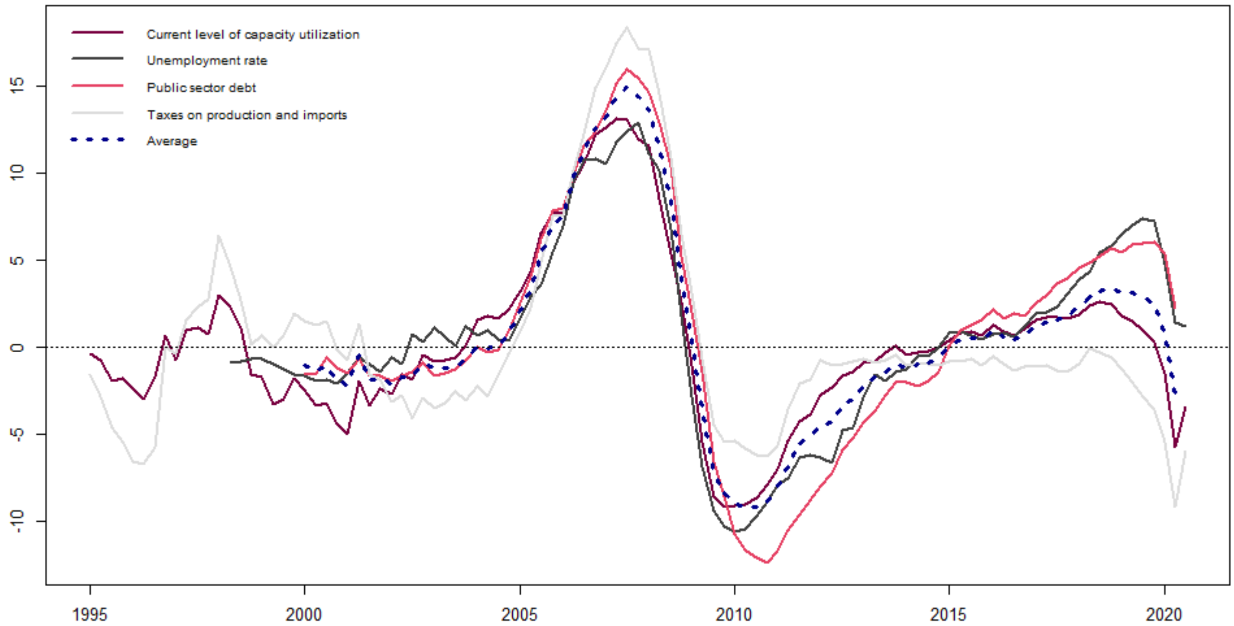


Figure 11. Bivariate UC model estimates and their average, Latvia's data

Similarly to Lithuania, Estonia's bivariate model estimates of GDP and current level of capacity utilization model depicted in Figure E.9. follow a similar pattern as the univariate output gap model but have sharper more distinct peaks. The model with current account balance (Figure E.10.) results in a classical business cycle estimate with very distinct periods or approximately 5 years and well-defined peaks. The bivariate model involving unemployment rate (Figure E.11.) is similar to the one of Latvia (Figure E.6.) resulting in a prolonged period cycle. The last model involving Compensation of employees in Figure E.12. is similar to the models with taxes on production and imports variable of Lithuania and Latvia. The similarity comes from the fact that the labour income results in a great portion of the tax revenue. Figure 12 displays all four bivariate models and their averaged value. The cyclical patterns are similar to the ones of Lithuania's data but differences between cycle estimates of each variable are greater.

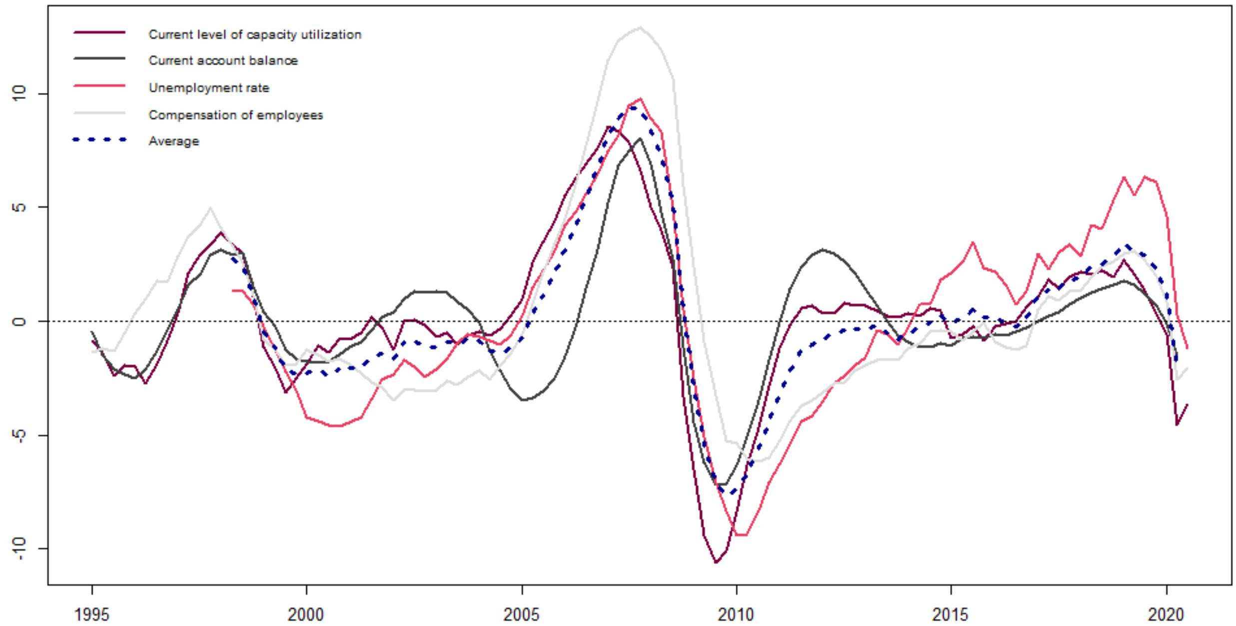


Figure 12. Bivariate UC model estimates and their average, Estonia's data

When overviewing all bivariate models for all three Baltic countries, one can notice the different impacts of each selected variable. Current level of capacity utilization has proven to be effective in refining the output gap, extracting a more distinctive pattern with smooth but well-defined peaks. In the bivariate model between Estonia's GDP and current account balance, the extracted cycle has a resemblance to a classical business cycle with short periods and smooth pattern thus this variable definitely be considered when estimating the output gap. Other variables are useful in amplifying cycle peaks and extracting longer business cycle. The models involving fiscal variables such as taxes on production and imports might be useful for fiscal policy makers when making policy decisions.

CONCLUSIONS

The thesis discussed the applied aspects of the potential output and the output gap estimates for quarterly GDP data of Baltic states that were obtained using two conventional statistical filters (Hodrick-Prescott and Christiano-Fitzgerald), univariate unobserved components model and bivariate unobserved components model which includes additional macroeconomic variables. The obtained results are generalized into the following five conclusions:

1. The Hodrick-Prescott filter is applied using the optimised value of the smoothing penalty parameter λ for each country that allows obtaining more country-specific estimates. The extracted cycles indicate a similar cyclical pattern for all three Baltic countries, with Latvia having nosier cycle than Lithuania and Estonia. The pitfalls below potential indicating various economic recessive cycles are indicated. One can observe the end of the sample problem specific to the HP cycle, where the value of the output gap is largely driven by the actual observed output. The mean of the cycle estimates is assessed as technical zeroes indicating that all three countries have been in both the recessionary and the expansionary cycle for around the same amount of time for the observed period.
2. The Christiano-Fitzgerald filter is applied with specified upper and lower cut-off cycle lengths of a minimum of 6 quarters and a maximum of 40 quarters. The CF filter estimates follow a similar pattern as the ones obtained with HP filter. The trend is more volatile resulting in a much smoother cycle than HP cycle. The cycle peak values have higher amplitude compared to HP cycle and a greater mean value.
3. Univariate unobserved components model used in the analysis follows a specification of the assumed stochastic and damped cycle and smooth trend, where the variance of the trend component is assumed to be zero ($\sigma_\eta^2 = 0$). The specification differs only for Latvia, where the cycle was assumed to be stochastic with the dampening factor restricted to one. The period cycle bounds are set to be between 6 and 48 quarters. The estimates of the output gap obtained using univariate UC model follow a similar pattern to the ones exhibited by statistical filters but within a smaller deviation amplitude from the potential output. Lithuania's and Estonia's estimates are closer to the HP filter ones in terms of the cycle pattern and variability and Latvia's cycle follows a pattern closer to the CF filter estimates. Periods of the cycles are estimated to be approximately 5 years. Such value might indicate an impact of political cycles – the change in the ruling government often brings changes in the economic system as well.
4. Together with univariate UC model, a "Beauty contest" variable selection based on three criteria is performed, which results in four variables being selected for each country out of

the starting nine. The selected variables for Lithuania are current level of capacity utilization, compensation of employees, net lending/borrowing and taxes on production and imports, for Latvia – current level of capacity utilization, unemployment rate, gross public sector debt and taxes on production and imports, and for Estonia – current level of capacity utilization, current account balance, unemployment rate and compensation of employees. The selection procedure shows the importance of pro-cyclical fiscal variables for Lithuania and Latvia but not Estonia, highlighting Estonia's fiscal stability.

5. Four bivariate unobserved component models with selected variables are obtained for each Baltic country, twelve in total. Additionally, an averaged cycle of all four bivariate models is obtained. In Lithuania's and Estonia's case, the averaged obtained cyclical pattern resembles one of the univariate models only with a smoother pattern, whereas Latvia has longer period cycles with fewer peaks. The bivariate models with current level of capacity utilization have performed very well in extracting a more distinctive pattern with smooth but well-defined peaks. Estonia's bivariate model estimate involving current account balance has resulted in a cycle resembling the classical business cycle with short periods and smooth pattern. Other variables are useful in amplifying cycle peaks and extracting the output gap with longer period cycles. The models involving fiscal variables might be useful for fiscal policy makers when making policy decisions. The averaged cycle estimate might be an alternative solution which highlights the impact on all four selected variables. Graphical comparison of each country's HP, CF, UCM and averaged bivariate UCM cycle estimates are given in Appendix F.

The future research could involve adjusting the univariate UC model specifications for each variable in a way that cycle extraction would be successful, essentially eliminating the need of the first criterion. This would involve adjusting cycle period bounds, specifying cycle and trend components. Another suggested improvement would be moving from the bivariate UC model into the multivariate UC setting, combining all selected variables which would allow the introduction of additional economic relationships .

BIBLIOGRAPHY

1. Alichi A., Bizimana O., Domit S., Fernandez Corugedo E., Laxton D., Tanyeri K., Wang H., Zhang F., Multivariate Filter Estimation of Potential Output for the Euro Area and the United States, *IMF working paper*, WP/15/253, 2015.
2. Alvarez L. J., Gomez-Loscos A., A Menu on Output Gap Estimation Methods, *Banco de Espana Working Paper No. 1720*, 2017.
3. Ball L., Leigh D., Loungani P., Okun's Law: Fit at 50?, *IMF Working Paper WP/13/10*, 2013.
4. Bernhardsen T., Eitrheim O., Jore A., Roisland O., Real-time Data for Norway: Challenges for Monetary Policy, *North American Journal of Economics and Finance*, vol 16, 2005, pp. 333-349.
5. Brooks R., Basile G., Global Macro Views – Nonsense Output Gaps, Institute of International Finance, May 2019.
6. Cayen J.-P., Van Norden S., The Reliability of Canadian Output Gap Estimates, *North American Journal of Economics and Finance*, vol 16, 2005, pp. 373-393.
7. Cerra V., Chaman Saxena S., Alternative Methods of Estimating Potential Output and the Output Gap: an Application to Sweden, *IMF Working Paper*, WP/00/59, 2000.
8. Chiu A., Wieladek T., Did Output Gap Measurement Improve Over Time?, *No 36, Discussion Papers*, Monetary Policy Committee Unit, Bank of England, 2012.
9. Cotis J.-P., Elmeskov J., Mourougane A., Estimates of Potential Output: Benefits and Pitfalls from a Policy Perspective, OECD Economics Department, 2004.
10. Cuerpo C., Cuevas A., Quilis E. Q., Estimating Output Gap: a Beauty Contest Approach, *AIReF Working Paper No. WP/2018/2*, 2018.
11. Darvas Z., Simon A., Filling the Gap: Open Economy Considerations for more Reliable Potential Output Estimates, *IEHAS Discussion Papers*, No. MT-DP - 2015/57, Hungarian Academy of Sciences, Institute of Economics, Budapest, 2015.
12. Doménech R., Gómez V., Estimating Potential Output, Core Inflation, and the NAIRU as Latent Variables, *Journal of Business and Economic Statistics*, 24(3), 2006, p.p. 354-365.
13. Fedderke J. W., Mengisteab D. K., Estimating South Africa's Output Gap and Potential Growth Rate, *ERSA Policy Paper 21*, March 2016.
14. Frale C., De Nardis S., Which Gap? Alternative Estimations of Potential Output and the Output Gap in the Italian Economy, Ufficio Parlamentare di Bilanco, *Working Paper no. 2*, July 2017.

15. Grigoli F., Herman A., Swiston A., Di Bella G., Output Gap Uncertainty and Real-time Monetary Policy, *Russian Journal of Economics*, vol 1 (4), 2015, pp. 329-358.
16. Harvey, A.C. & Jaeger, A., Detrending, Stylized Facts and the Business Cycle, *Journal of Applied Econometrics,* Vol. 8, No. 3, Jul-Sep., 1993, pp. 231-247
17. Havik K., Mc Morrow K., Orlandi F., Planas C., Raciborski R., Röger W., Rossi A., Thum-Thysen A., Vandermeulen V., The Production Function Methodology for Calculating Potential Growth Rates & Output Gaps, *European Economy, Economic Papers no. 535*, November 2014.
18. Ho G., Mauro P., Growth: Now and Forever?, *IMF Working Paper*, 14/117, 2014.
19. Hodrick R., Prescott E. C., Post War Business Cycles: An Empirical Investigation, *Journal of Money Credit and Banking*, 1-16, 1997.
20. Kuttner K. N., Estimating Potential Output as a Latent Variable, *Journal of Business and Economic Statistics*, vol 12(3), 1994, p.p. 361-368.
21. Lancaster D., Tulip P., Okun’s Law and Potential Output, *Reserve Bank of Australia Research Discussion Paper No. 2015-14*, 2015.
22. Ley E., Misch F., Real-time Macro Monitoring and Fiscal Policy, *Policy Research Working Paper 6303,* Washington, DC: World Bank, 2013.
23. Murray J., Working paper No. 5 Output Gap Measurement: Judgement and Uncertainty, Office for Budget Responsibility, July 2014,
24. Navarni L., Zoppe A., Potential Output Estimated and their Role in the EU Fiscal Policy Surveillance, European Parliament, 2020
25. Nelson E., Nikolov K., UK Inflation in the 1970s and 1980s: the Role of Output Gap Mismeasurement, *Journal of Economics and Business,* vol. 55(4), 2003, pages 353-370.
26. Okun A., Potential GNP: Its Measurement and Significance, Cowles Foundation, Yale University, 1962
27. Ollivaud P., Turner D., The Effect of the Global Financial Crisis on OECD Potential Output, *OECD Economics Department Working Papers No. 1166*, OECD Publishing, 2014.
28. Orphanides A., Monetary Policy Rules Based on Real-time Data, *American Economic Review*, vol. 91, 2001, pp. 964–985.
29. Orphanides A., van Norden S., The Unreliability of Output- Gap Estimates in Real Time, *Review of Economics and Statistics*, MIT Press, vol 84 (4), 2002, pp. 569-583.
30. Pedersen T., Alternative Linear and Non-Linear Detrending Techniques: A Comparative analysis based on Euro-Zone Data, Copenhagen: Ministry of Economic and Business Affairs, (2002, November).

31. Tosetto E, Revisions of Quarterly Output Gap Estimates for 15 OECD Countries, *OECD Working Paper*, 2008.

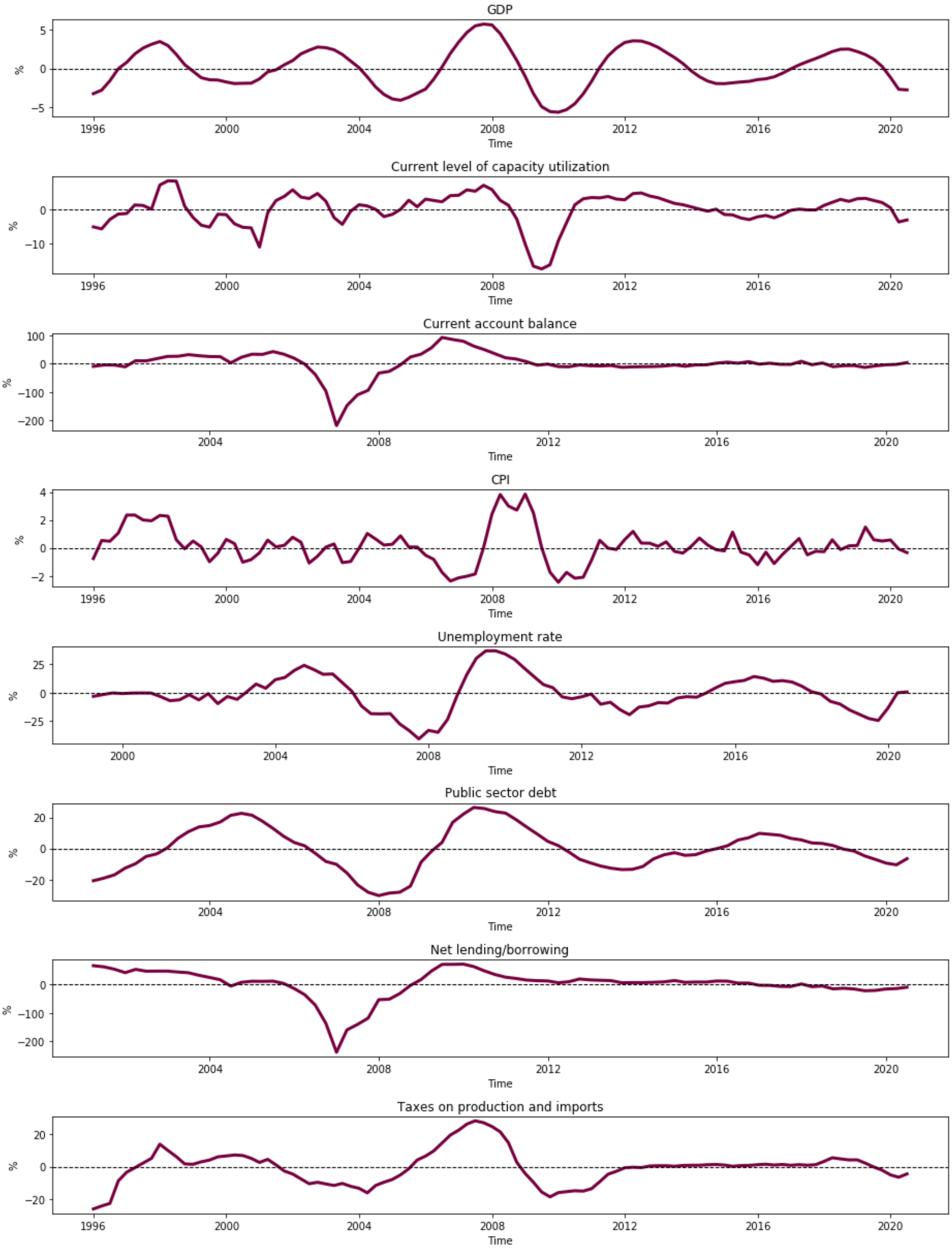
APPENDIX A. Necessary and sufficient selection criteria[10]

Criterion	Description
N_1	Statistical significance of the coefficients, focusing on the loadings of the observables on the cycle.
N_2	Average relative revision, defined as the average distance between one-sided and two-sided estimates, relative to the maximum amplitude of the output gap estimate.
N_3	Average relative uncertainty surrounding the cycle estimates, as the average standard error relative to the maximum amplitude.
S_1	Economic soundness, meaning that some key macroeconomic relationships could be captured by variables if included in the model (e.g. Okun's Law, Phillips Curve, etc.).
S_2	The amplitude and profile alignment with consensus figures (range given by a panel of official institutions) and in agreement with commonly accepted business cycle chronology (e.g. ECRI dating). The quantification of the profile alignment can be made employing the cross-correlation function and different measures of conformity.
S_3	Stability of the one-sided cycle estimate, as this would mimic the practitioner's need for updated estimates as new data is added in real-time. Stability can be measured using the revisions of the one-sided estimates.

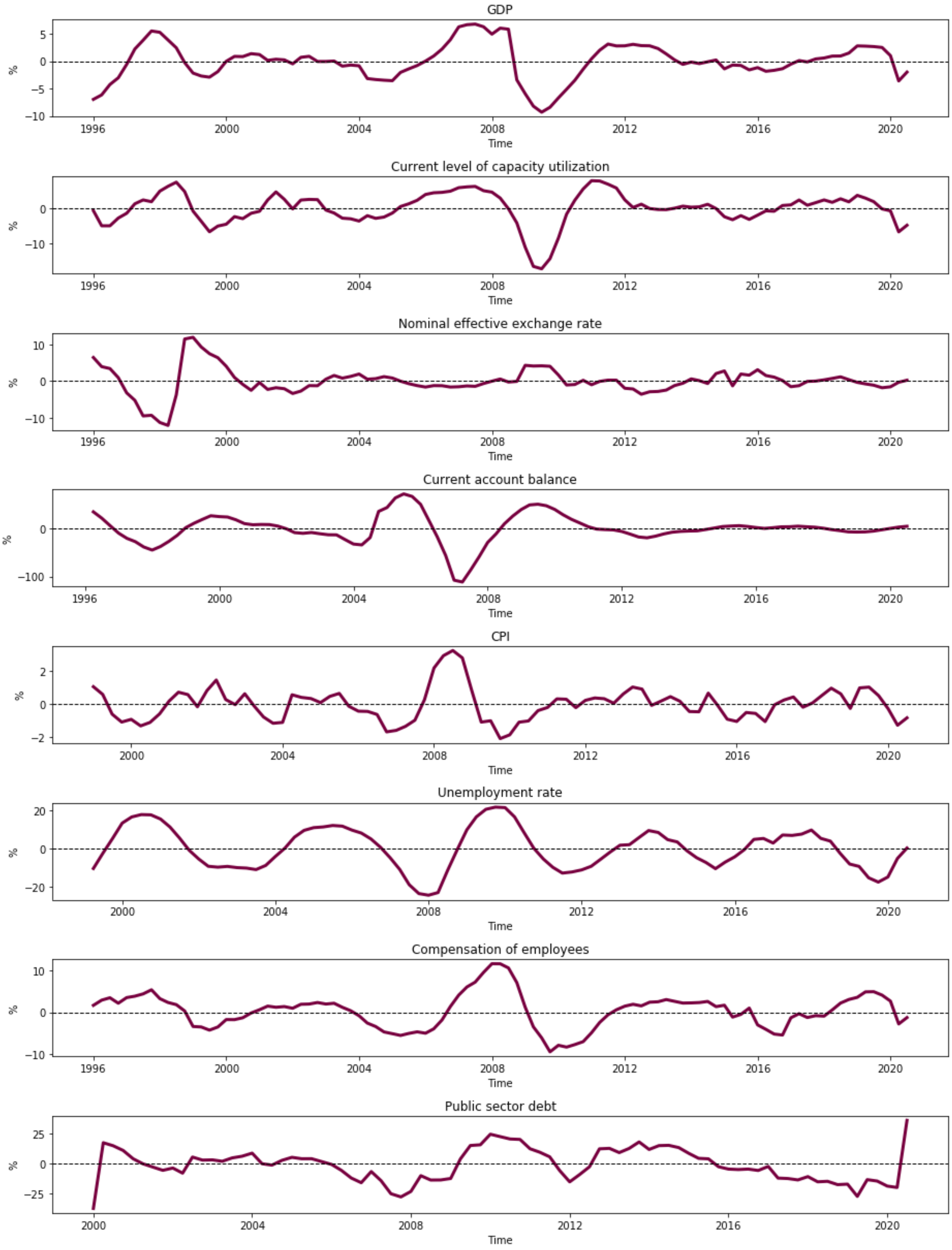
APPENDIX B. Descriptive statistics of the data set

Variable	Mean	Median	Minimum	Maximum	Std.Dev	Skewness	Kurtosis
LT GPD at market prices	96.79	99.90	51.00	141.40	26.91	-0.16	-1.23
LV GDP at market prices	98.67	106.00	53.20	134.70	25.91	-0.39	-1.25
EE GDP at market prices	98.52	103.70	52.20	140.70	25.56	-0.29	-1.06
LT Current level of capacity utilization	66.04	70.00	44.10	77.80	10.21	-0.78	-0.71
LV Current level of capacity utilization	66.48	70.40	48.40	77.40	8.12	-0.76	-0.76
EE Current level of capacity utilization	70.05	72.30	55.50	79.40	6.25	-0.87	-0.18
LT Nominal effective exchange rate - 42 trading partners (industrial countries)	91.05	97.08	34.27	124.96	24.78	-1.08	0.25
LV Nominal effective exchange rate - 42 trading partners (industrial countries)	101.35	103.54	66.15	122.62	14.16	-1.16	0.69
EE Nominal effective exchange rate - 42 trading partners (industrial countries)	95.78	97.42	65.99	117.35	13.56	-0.82	0.07
LT Current account balance	-4.75	-4.22	-19.61	8.55	6.06	-0.17	-0.52
LV Current account balance	-4.77	-4.83	-24.22	11.38	7.33	-0.72	0.66
EE Current account balance	-4.58	-4.14	-17.56	4.75	6.21	-0.36	-1.21
LT CPI	83.91	82.17	43.61	111.44	17.67	-0.09	-1.18
LV CPI	78.52	83.11	36.64	109.44	22.71	-0.17	-1.58
EE CPI	84.33	81.96	54.62	110.38	17.95	-0.18	-1.46
LT Unemployment rate	10.89	9.50	4.10	18.20	4.12	0.09	-1.16
LV Unemployment rate	11.47	10.50	5.50	20.50	3.72	0.44	-0.31
EE Unemployment rate	9.04	7.90	4.00	18.30	3.51	0.54	-0.41
LT Compensation of employees	2722.77	2864.20	442.00	6081.20	1518.70	0.35	-0.79
LV Compensation of employees	1883.84	1932.10	419.30	3929.40	1035.41	0.25	-1.15
EE Compensation of employees	1648.50	1736.70	347.60	3517.60	919.10	0.35	-0.93
LT Gross Public Sector Debt, General Gov.	34.09	33.34	10.35	46.79	10.28	-0.55	-0.82
LV Gross Public Sector Debt, General Gov.	32.80	17.60	13.00	54.20	15.57	-0.13	-1.89
EE Gross Public Sector Debt, General Gov.	10.12	8.30	3.90	23.30	3.28	0.51	1.39
LT Net lending/borrowing (current and capital account)	-3.28	-3.90	-15.04	8.51	6.66	0.01	-1.24
LV Net lending/borrowing (current and capital account)	-3.21	-3.83	-23.71	12.39	7.62	-0.69	0.41
EE Net lending/borrowing (current and capital account)	-3.30	-3.23	-16.50	9.08	6.96	-0.16	-1.36
LT Taxes on Production and Imports	762.19	799.90	148.90	1443.80	376.43	0.12	-1.13
LV Taxes on Production and Imports	556.78	556.00	130.00	1103.60	299.30	0.24	-1.20
EE Taxes on Production and Imports	478.46	485.90	88.40	1037.50	273.35	0.26	-1.11

APPENDIX C. Univariate unobserved components model cycle estimates of Latvia's data



APPENDIX D. Univariate unobserved components model cycle estimates of Estonia's data



APPENDIX E. Bivariate unobserved components model estimates

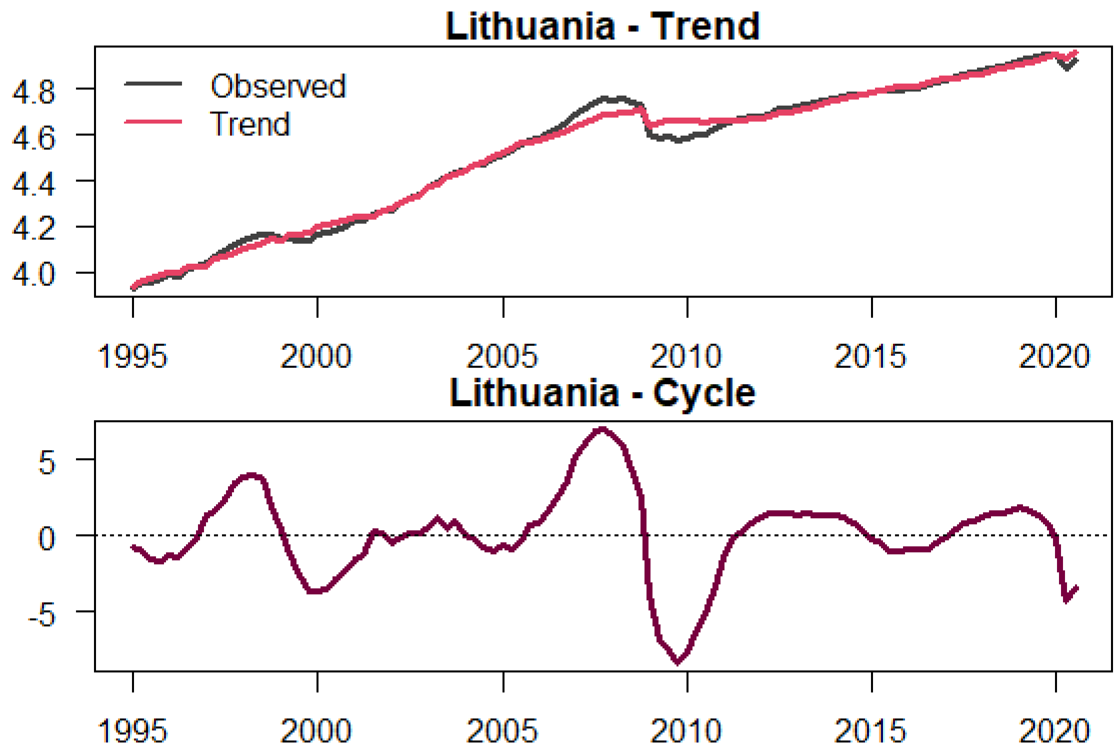


Figure E.1. Bivariate UC model for GDP and current level of capacity utilization, Lithuania's data

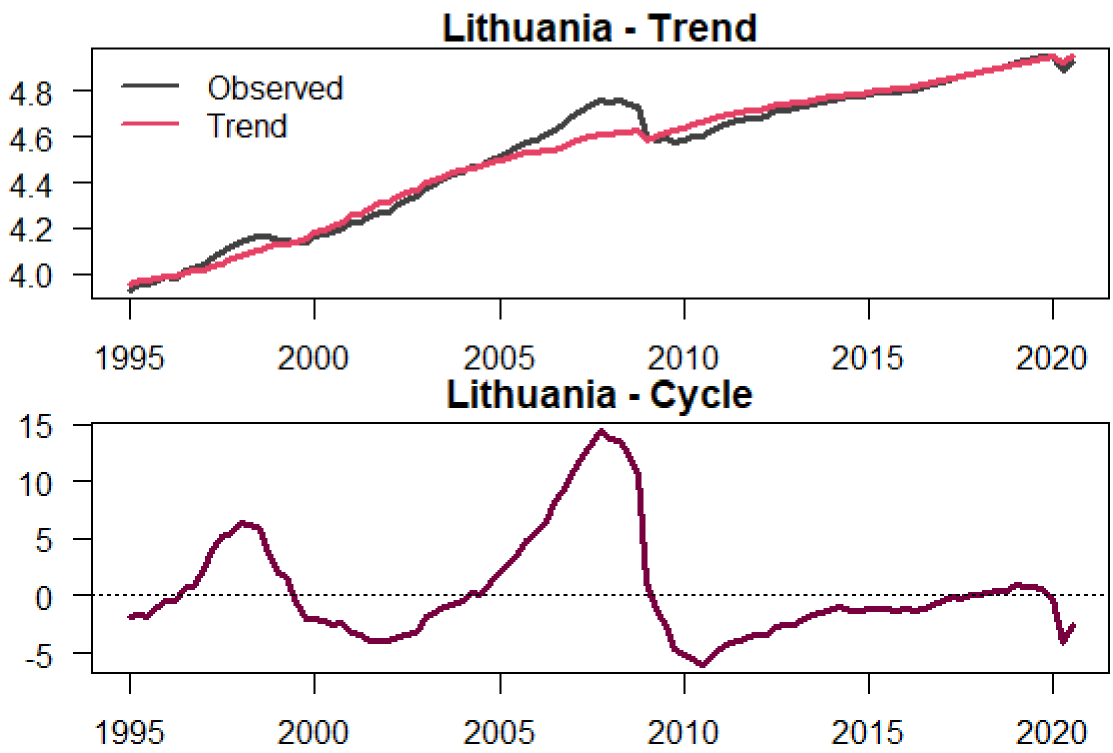


Figure E.2. Bivariate UC model for GDP and compensation of employees, Lithuania's data

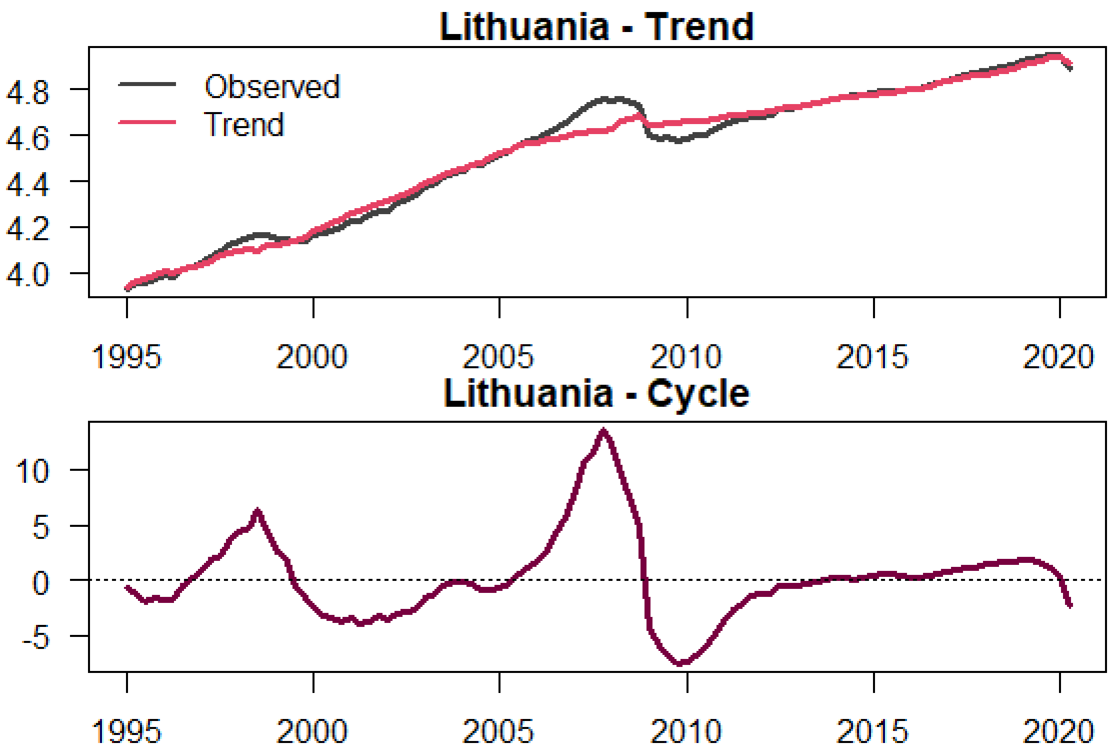


Figure E.3. Bivariate UC model for GDP and net lending/borrowing, Lithuania's data

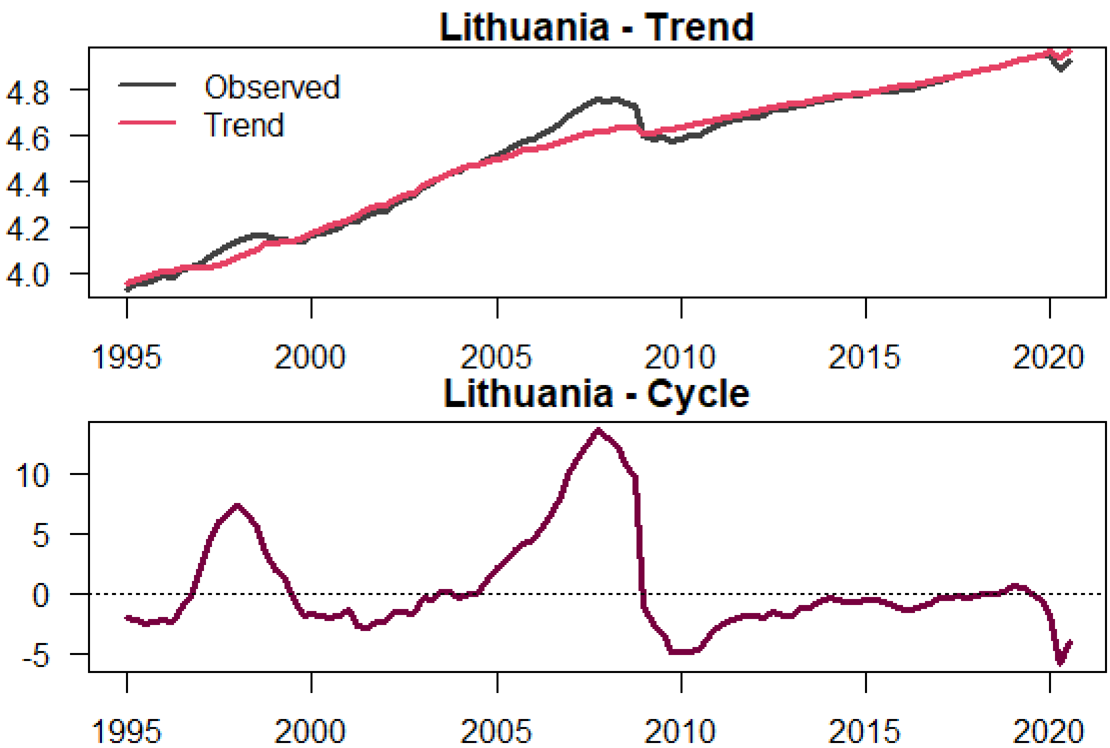


Figure E.4. Bivariate UC model for GDP and taxes on production and imports, Lithuania's data

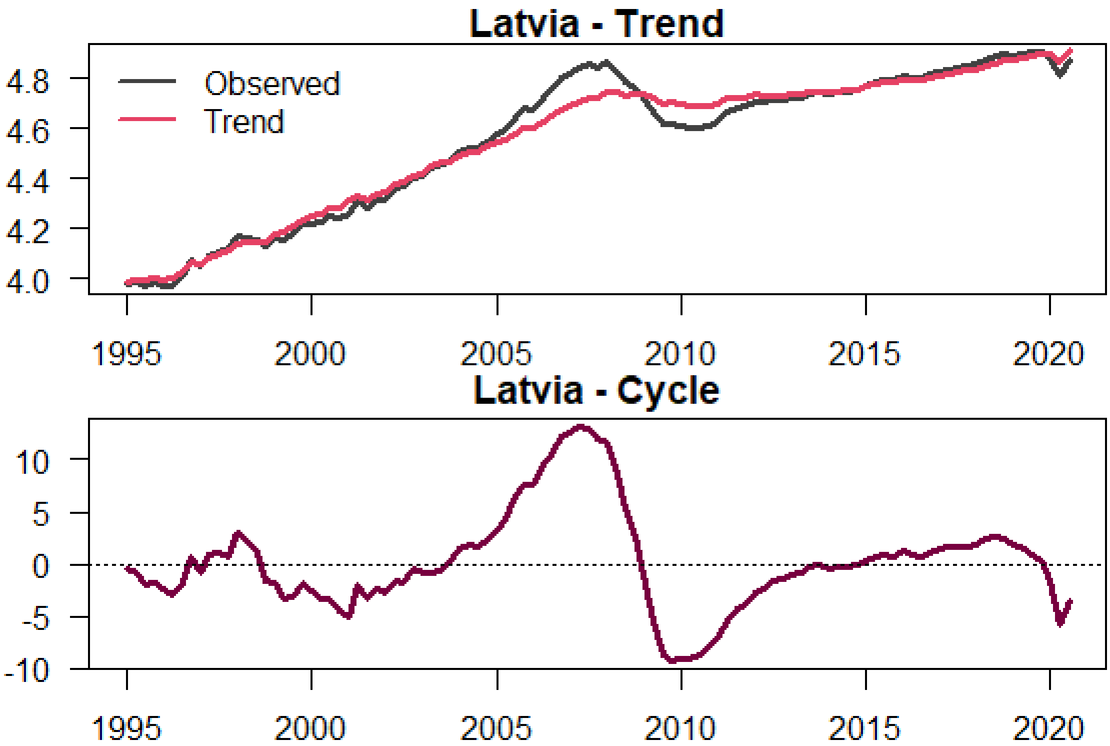


Figure E.5. Bivariate UC model for GDP and current level of capacity utilization, Latvia's data

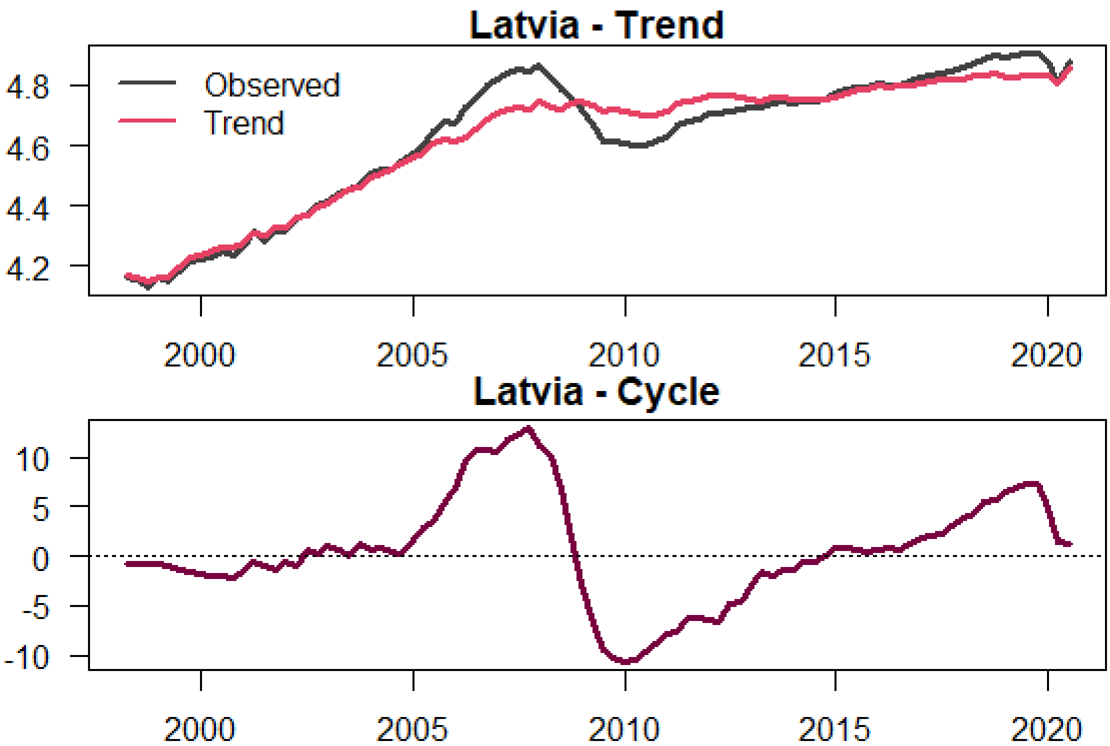


Figure E.6. Bivariate UC model for GDP and unemployment rate, Latvia's data

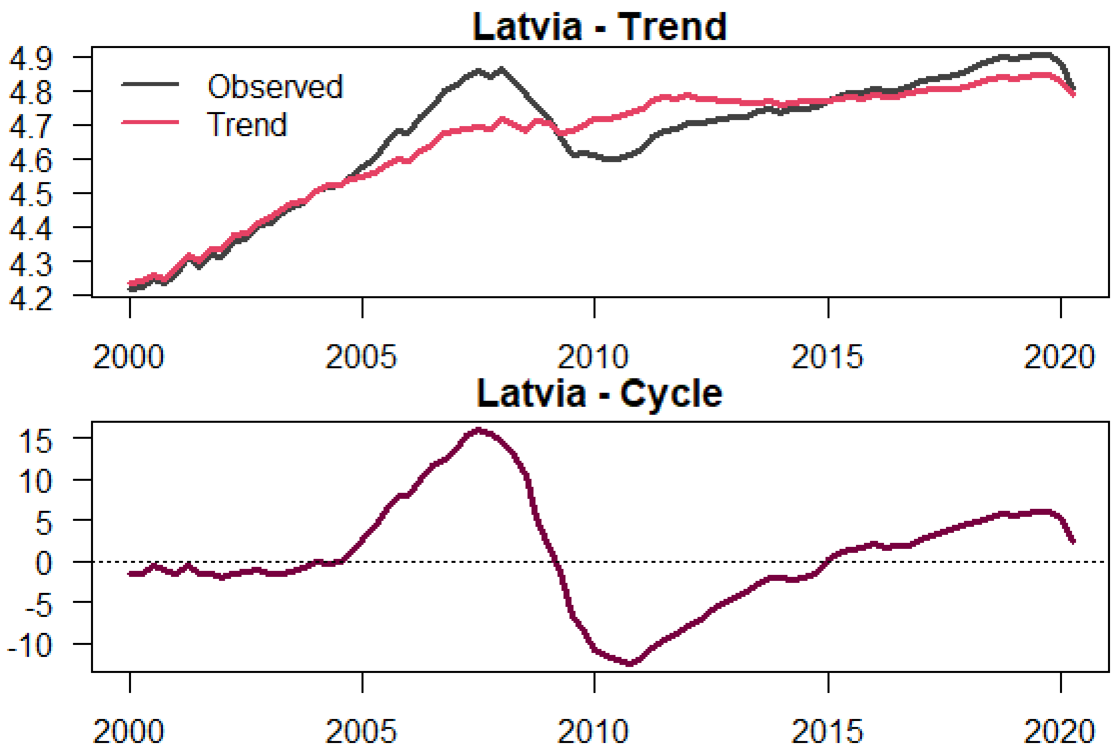


Figure E. 7. Bivariate UC model for GDP and gross public sector debt, Latvia's data

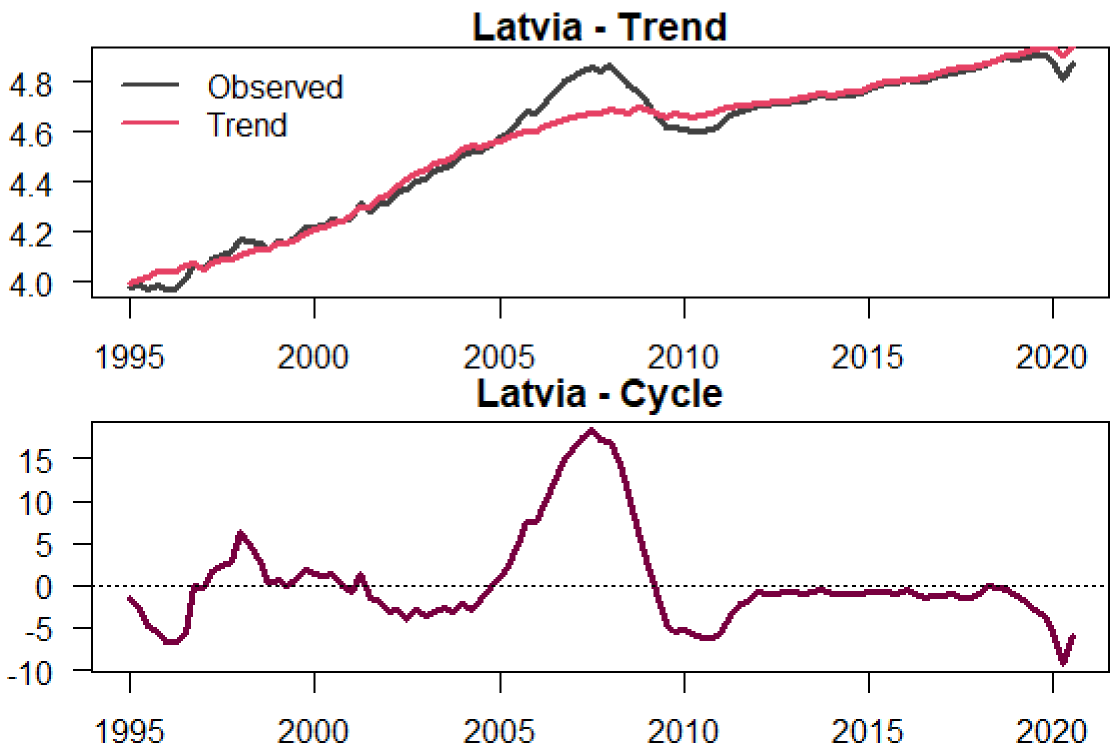


Figure E.8. Bivariate UC model for GDP and taxes on production and imports, Latvia's data

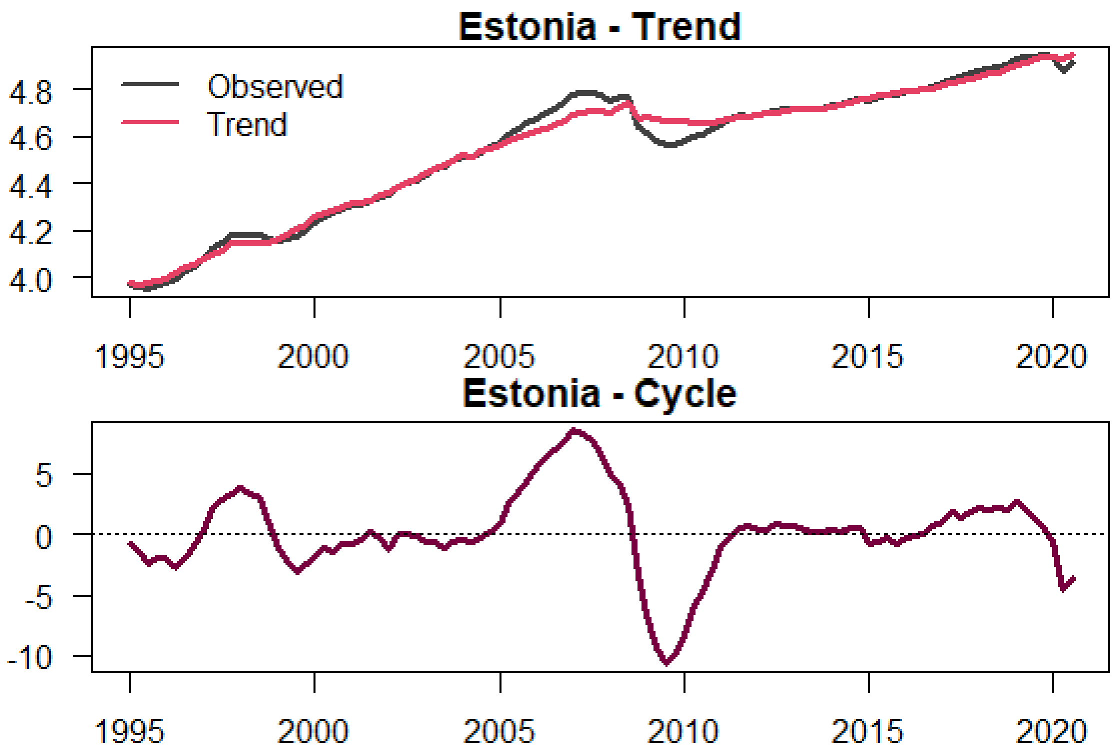


Figure E.9. Bivariate UC model for GDP and current level of capacity utilization, Estonia's data

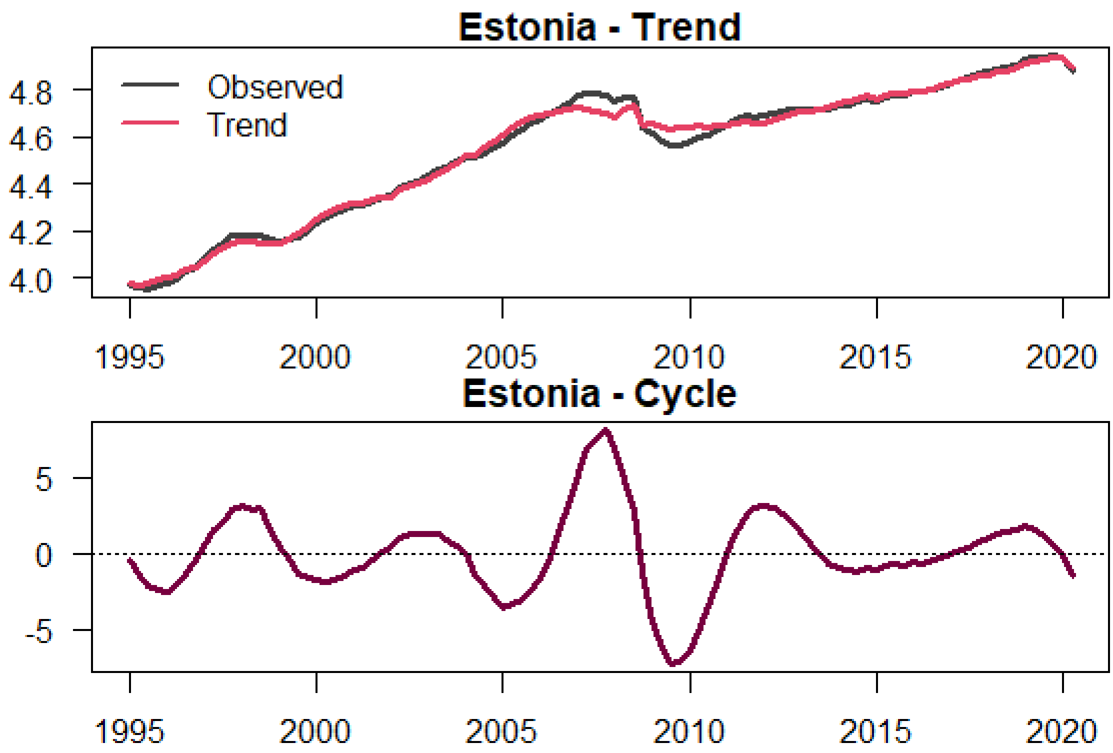


Figure E.10. Bivariate UC model for GDP and current account balance, Estonia's data

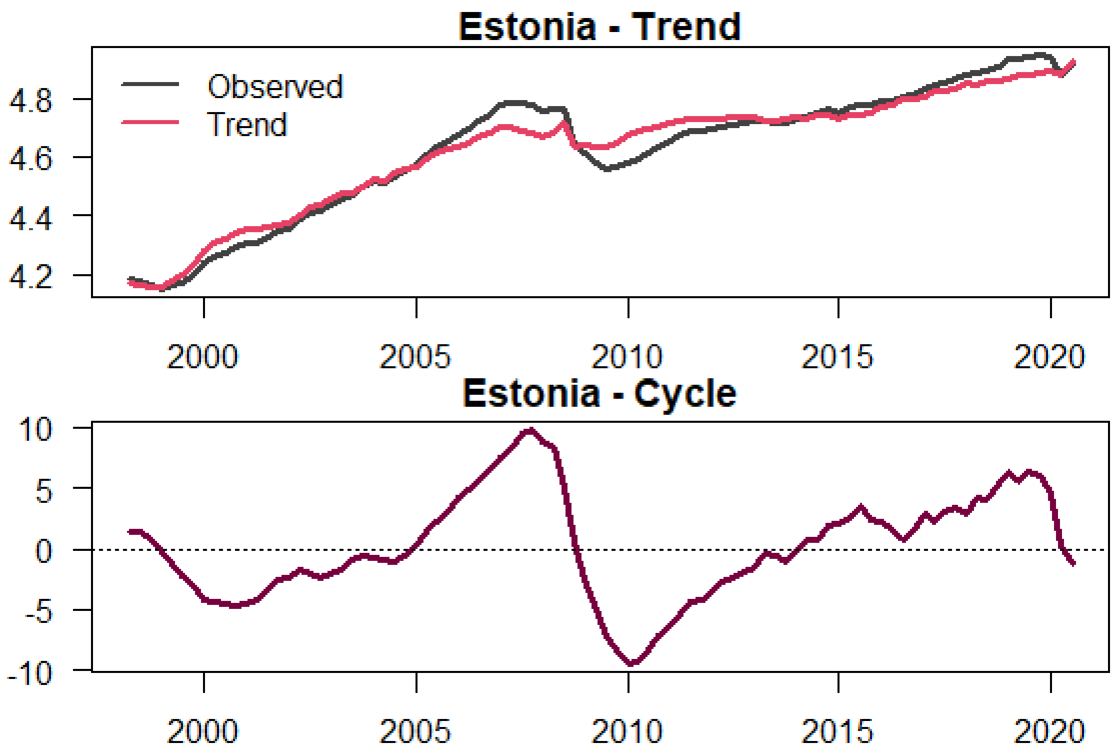


Figure E.11. Bivariate UC model for GDP and unemployment rate, Estonia's data

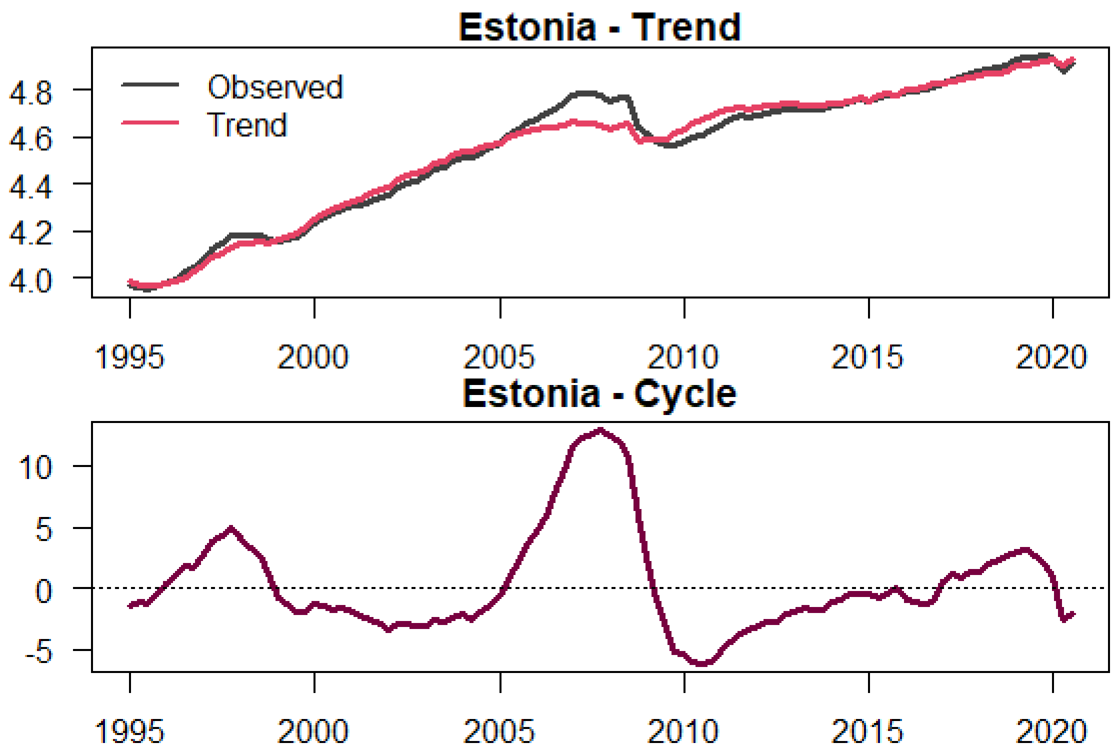


Figure E.12. Bivariate UC model for GDP and compensation of employees, Estonia's data

APPENDIX F. Method comparison

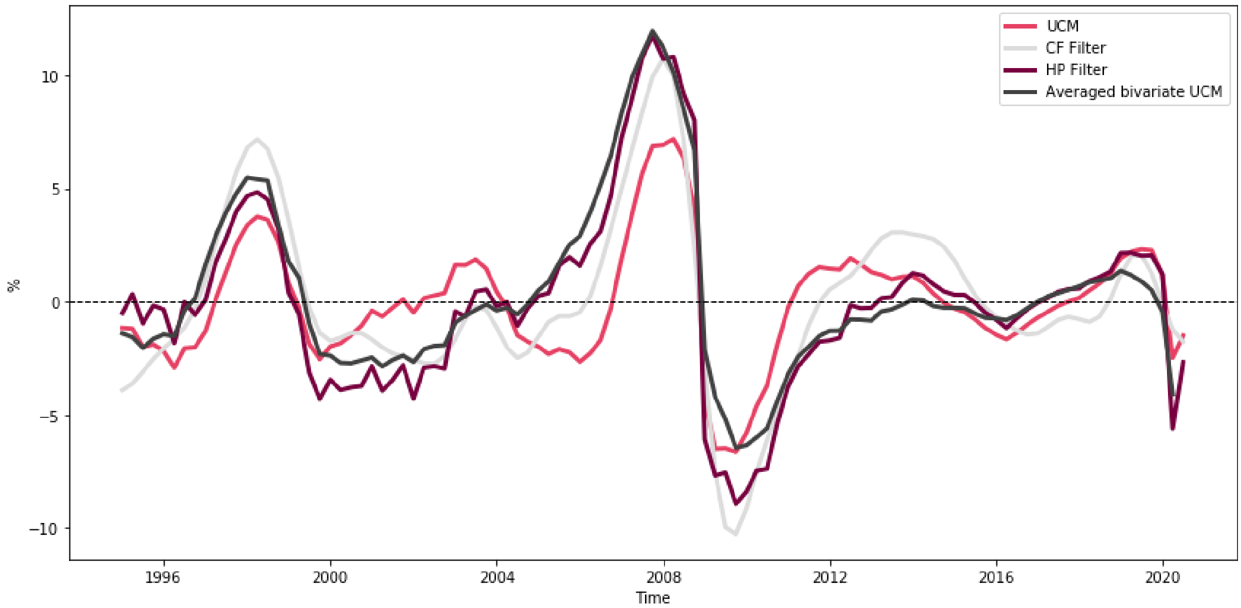


Figure F.1. Method comparison for Lithuania’s data

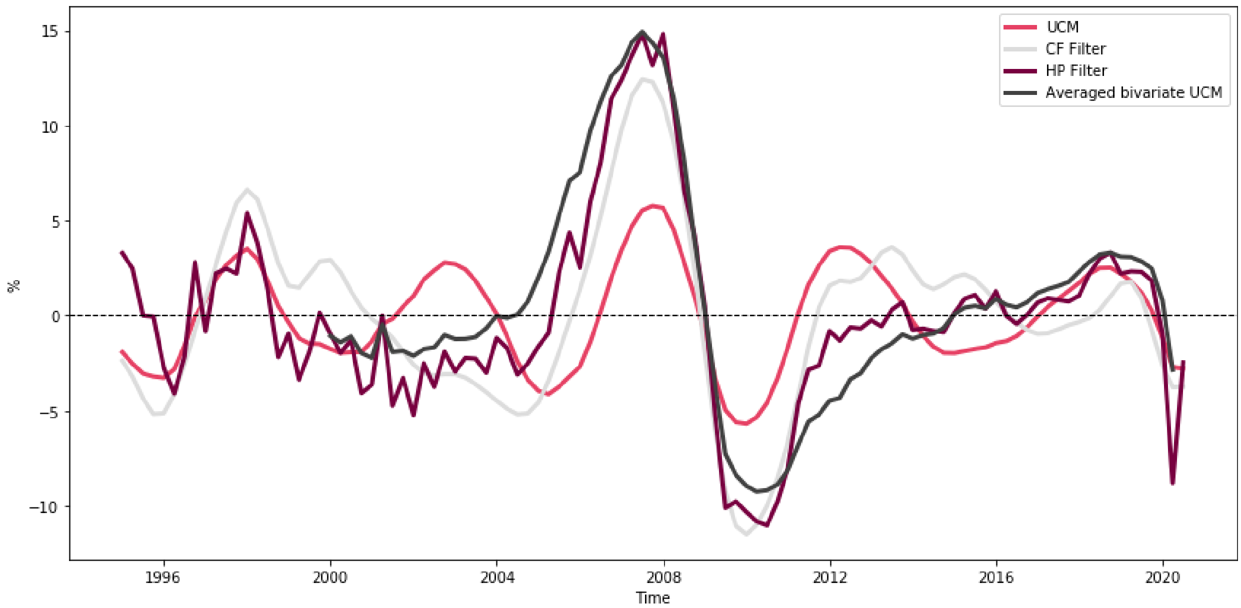


Figure F.2. Method comparison for Latvia’s data

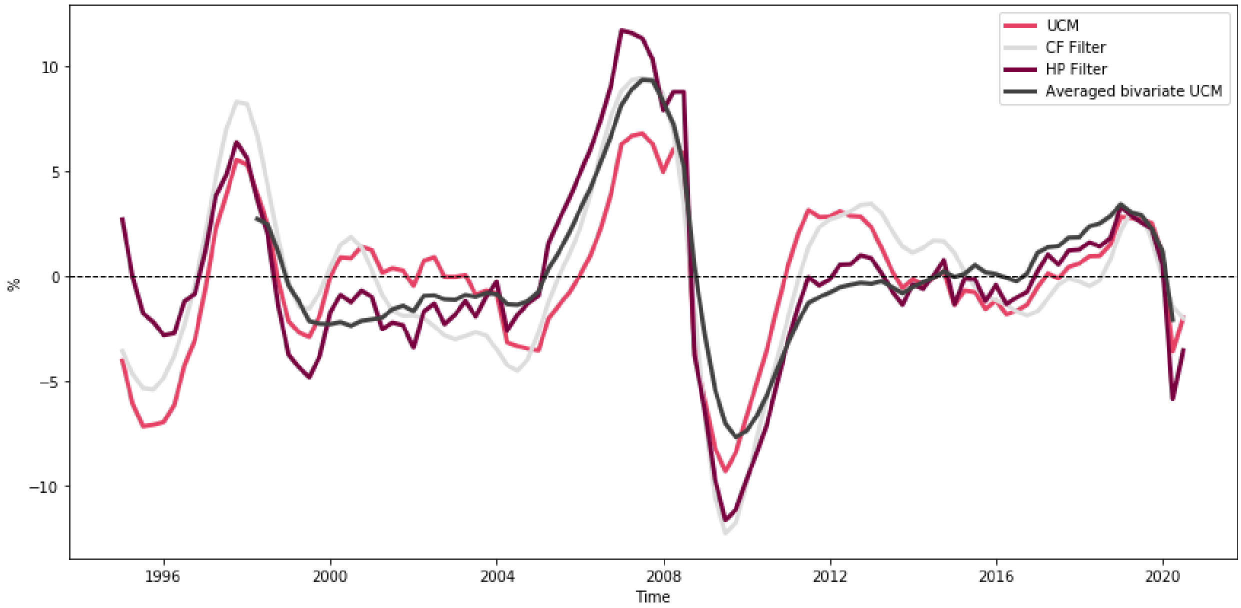


Figure F.3. Method comparison for Estonia's data

APPENDIX G. Smoothing parameter λ optimization for Hodrick-Prescott filter

The spectral representation theorem implies the existence of an ideal infinite-dimensional low-pass filter that in the time-domain follows an infinite two-sided moving average:

$$y_t^f = \sum_{i=-\infty}^{\infty} h_i^f \, y_{t-i}, \quad \sum_{i=-\infty}^{\infty} \left|h_i^f\right| < \infty, \; h_0^f = \frac{\omega_f}{\pi}, \; h_i^f = \frac{\sin\left(i\omega_f\right)}{i\pi}, \omega_f = \frac{2\pi}{p_f}, f \in \{l, u\}, \tag{G.1}$$

The difference of two ideal low-pass filters defined at distinct cut-off frequencies determines an ideal band-pass filter with the coefficients $h_i^{bp} = h_i^l - h_i^u$, while an ideal high pass-filter has $h_i^{hp} = \mathbb{I}\{i = 0\} - h_i^u$, where an indicator function $\mathbb{I}\{i = 0\}$ equals to one, when $i = 0$, and to zero elsewhere.

In order to achieve optimal λ value, analysis of the power transfer function is concluded. Any zero-mean stationary time series admits equivalent definition by autocovariance function $\gamma(\tau) = \mathbb{E}(y_t y_{t-\tau}')$ or a spectral density:

$$S_y(\omega) = \frac{1}{2\pi} \sum_{\tau=-\infty}^{\infty} e^{-i\omega\tau} \gamma(\tau). \tag{G.2}$$

The ideal filter (G.1) or its finite-dimensional approximation defines the transfer function $h\left(e^{-i\omega}\right)$ and the link between the initial spectral density and the spectral density of filtered series $S_c(\omega)$:

$$S_c(\omega) = H(\omega) \cdot S_y(\omega) = \left|h\left(e^{-i\omega}\right)\right|^2 \cdot S_y(\omega), \tag{G.3}$$

where $H(\omega)$ is the sought power transfer function (PTF). PTF fully describes the change in the relative importance of the cyclical components in y_t^f: $H(\omega) > 1$ amplifies the amplitude of the cycle corresponding to the frequency ω, $H(\omega) < 1$ dampens the amplitude of the respective cycle, and $H(\omega) = 1$ keeps the amplitude unchanged. Such amplification and dampening may lead to spurious cycles or mute the important cyclical signal. In general, the filters will introduce two types of distortions:

1. Compression – a distortion inside the ideal filter PTF, when a part of band associated frequencies is lost;
2. Leakage – a distortion outside the ideal filter PTF, when a part of low-frequency data and the noise go to the cycle or within band frequencies are falsely amplified.

Quantifying the integral sum of distortions, Pedersen (2002) suggests minimizing the Q function by solving for the unknown parameters of the TCD method:

$$Q = \sum_{\omega \in \Omega} |S_c(\omega) - S_c^*(\omega)| \cdot \Delta\omega = \sum_{\omega \in \Omega} |H(\omega) - H^*(\omega)| \cdot S_y(\omega) \cdot \Delta\omega, \quad \text{(G.4)}$$

where $S_c^*(\omega)$ and $H^*(\omega) = \mathbb{I}\{\omega_u \leq \omega \leq \omega_l\}$ denote the spectral density of the filtered series and PTF of the ideal band-pass filter, while $S_c(\omega)$ and $H(\omega)$ indicate its finite-dimensional approximation.

The impact of different values of λ best shown in the frequency domain, analysing the power transfer functions of the trend and the cyclical component for different values of the penalty parameter.

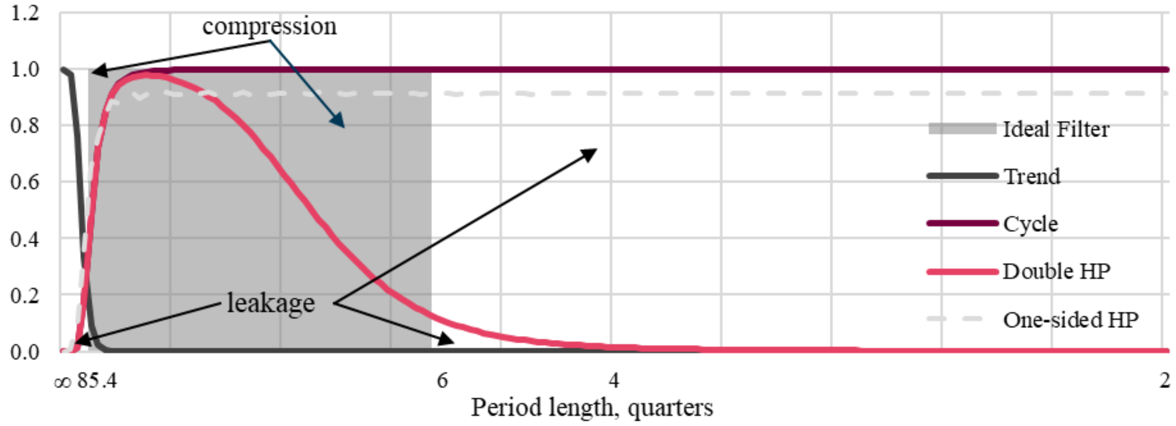


Figure G.1. Power transfer functions for the HP filter

Figure G.1 depicts the PTF of the HP filtered cycle determined by:

$$H^{HP}(\lambda, \omega) = \left[\frac{4(1-cos(\omega))^2}{4(1-cos(\omega))^2 + \frac{1}{\lambda}}\right]^2. \quad \text{(G.5)}$$

Since lower frequencies would be ideally allocated to the trend and higher frequencies to the cycle, higher values of λ shift the PTF of trend closer to zero implying smoother trends and approaches a linear trend in the limit. On the contrary, with lower values of λ, the trend becomes more volatile as it will contain more of the high-frequency spectrum approaching the original data when the penalty value drops to zero [30].