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Master thesis

Econometrical Marketing Mix Models
Ekonometriniai mišrios rinkodaros modeliai

Akvilė Mazanauskaitė

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Darbo vadovas Prof. Habil. Dr. Remigijus Leipus
Darbo recenzentas Prof. Habil. Dr. Alfredas Račkauskas

Darbas apgintas ____________________

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Ekonometriniai mišrios rinkodaros modeliai

Santrauka

Magistro baigiamajame darbe yra nagrinėjami mišrios rinkodaros modeliai. Šio darbo tikslas yra įrodyti modelio stabilumą ir įrodyti, kad optimizuojant R-kvadratą sprendžiant optimizavimo užduotį gauti parametrai yra stabilūs bei pastovūs. Taip pat buvo svarbu rasti trumpo ir ilgo laikotarpio investicinę grąža. Tuo tikslu buvo kuriamas vieno iš senesnių Lietuvoje kliento, turinčio fizines parduotuves, modelis, skirtas modeliuoti srautus į parduotuvę. 10 skirtingų medijų koeficientų ir 17 skirtingų kampanijų koeficientai buvo rasti optimizavimo būdu. Modelio rezultatai parodė, kad modelis gerai aprašo duomenis, yra stabilus bei patikimas. Taigi, šią metodologiją galima taikyti kitiems panašiems uždaviniams spręsti.

Raktiniai žodžiai: mišrios rinkodaros modeliai, S-kreivė, trumpalaikė investicinė grąža, ilgalaikė investicinė grąža.

Econometrical Marketing Mix Model

Abstract

In this thesis Marketing Mix Model methodology will be studied. The main purpose of this thesis is to prove model consistency and to prove that optimizing R^2, by solving multiple parameters, the procedure return consistent parameters. Also, there will be important to find short and long term of return of investments. In order to succeed goal of the thesis, one of the biggest brand with physical shops in Lithuania was examined. The coefficient for 10 different media and 17 different campaigns were found. The results prove model consistency and coefficients stability and show that these type of optimization problems could be further used.

Key words: Marketing Mix, S-shape, Ad-Stock, short term ROI, long term ROI.

Contents

Introduction

The marketing mix model is one of the most reliable way to evaluate Return on Investment (ROI) and to find optimal allocation of the marketing budget. The model structure is derived from microeconomic theories of consumer demand ranging from single equations of product sales to full interactive systems of brand choice. Then, econometric techniques are used to estimate demand response to marketing investments, separating product sales into base and incremental volume. Base sales are representing the long-run or trend component of the product time series, driven by factors ranging from regular shelf price and selling distribution to underlying consumer brand preferences. Incremental volume is essentially short-run in nature, capturing the weekly sales variation driven by temporary selling price, multibuy promotions and above the line media activity. These are converted into incremental revenues or profits and benchmarked against costs to calculate ROI to each element of the marketing mix.

Paying attention to incremental volumes in this way might imply that conventional marketing mix models provide insight only into short-term ROI. As, they often lead to marketing budget allocations biased towards promotional activity: short-run sales respond well to promotions, yet are less responsive to media activity – particularly for established brands. This, however, ignores the long-term perspective: that is, the potential brand-building effects of successful media campaigns on the one hand and the brand impacts of heavy price discounts on the other. Acknowledging and quantifying these features is crucial to a complete ROI evaluation and a more strategic budget allocation. Measuring the long-run impact of marketing investments requires a focus on the base sales component of the marketing mix model. This is simply because any long-term brand-building effects reside in the level or trend component of the sales series and impact the evolution in base sales over time. Used Econometric methodology helps to uncover these effects. The conventional approach uses static Ordinary Least Squares (OLS) techniques which impose a fixed or deterministic baseline. Not only can this give an artificial split into base and incremental volumes, it precludes any analysis of the long-run impact of marketing activity by construction.

So there is need to use a methodology that can directly separate both the short and long-run features of the data – allowing a complete analysis of both in distinct stages. Time series regression analysis is a one of the best choice for two reasons. Firstly, all marketing mix models involve time-ordered data and are essentially time series equations with additional marketing mix components. Secondly, the technique provides a direct decomposition of any time-ordered data series into a trend, seasonal and random error component. It is then a natural step to decomposition of product sales into short-term marketing factors (incremental) and long-term base (trend). This generates an evolving baseline, which can then be meaningfully analyzed to quantify long-run ROI.

Theory overview

Econometrical Marketing Models

Econometrical Marketing modeling process begins from the correct questions asked for model: what exactly econometrical should answer, which main KPI[1], should be used, if the needed data is available. When all these questions are answered then there is possibility to create proper model and evaluate total ROI. Cain [2] provides us Marketing Modeling Structure in Figure 1.

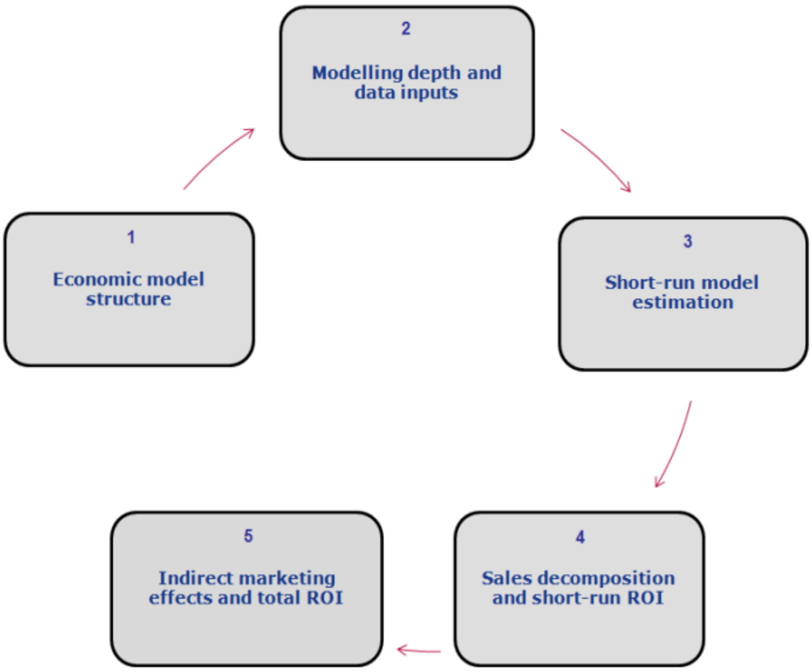


Figure 1: Econometrical Marketing Process

To evaluate short run ROI there is possibility from steps 1-4. These steps are standard Marketing Mix Modelling process. However, there is important to underlie correct microeconomic consumer demand structure which could be underlined by model form. Also there is important that standard Ordinary Least Square procedure helps us to evaluate short term ROI. By step 5 consumer tracking research is combined with output of step 1-4 helps us to evaluate long term ROI.

If there is a brand with only one marketing decision variable as advertising a descriptive model for this firm sales would be:

$$Q_t = f(A_t, Q_t), \tag{1}$$

where A_t -brand investment level, Q_t- environment actions.

The main problem that in this time there could be a lot of different type

[1]KPI- Key Performance Indicators

media investments so to avoid overfitting we need to summarize media investments from all media type. However, at the final there is need to know which media performs better. To solve this problem the optimization of R^2 is used. So there is searching coefficients for different media types to summarize all media investments, but in this time, there is possibility to know which media performed better from media coefficients.

Also, a lot of brand has great variety of different campaigns for the one brand. Some campaigns, usually, are for image, other are different type promotional campaigns and so on. The most times, several campaigns are running at the same time for the big brands. Then there is the important question: which campaign performed better? Also, to answer this question, there is need for the is possibility to solve optimization problem deeper by summarizing media type investments from the campaigns investments.

Summing it up, equation (1) optimization R^2 solves parameters for equation(2) and equation(3)

$$A_t = \alpha_1 A_{1t} + ... + \alpha_n A_{nt} + \epsilon_t, A_i \ - \ i^{th} \ media \tag{2}$$

$$A_{it} = \beta_1 C_{1t} + ... + \beta_m C_{mt}, C_j \ - \ j^{th} \ camapaign \ for \ i^{th} \ media \tag{3}$$

S-shapes

Perner [7] suggests that in many cases advertising leads to a relatively modest increase in sales. For example, that when a brand increases spendings to advertising by 1%, sales go up by 0.05%, in contrast, if prices are lowered by 1%, sales increases by 2%). So, advertising is more effective in selling durble goods than for non-durable goods. Perner [7] suggests that advertising effectiveness follows "S - shaped" curve:

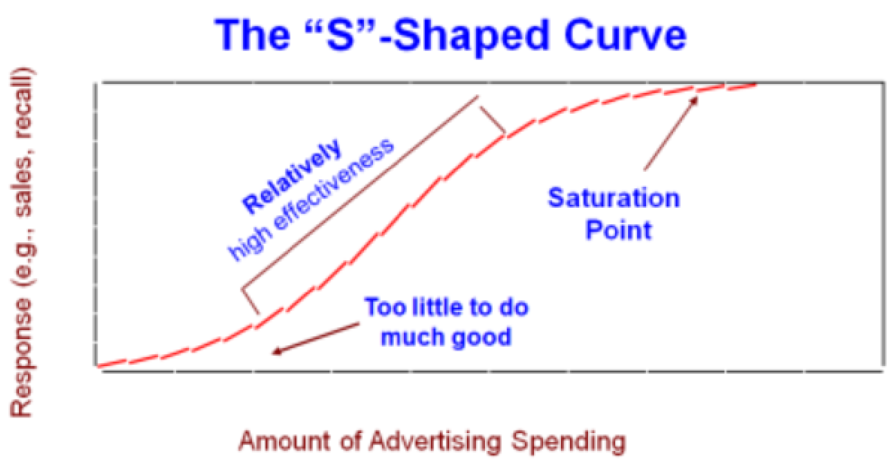


Figure 2: S-shape curve

From Figure 2 small amounts of advertising are uneffective, so it is hard to effect on customers. At the medium level, advertising may be effective. However, above saturation point additional adverting appears to have a limited effect.

Figure 2 is generated by equation(4)

$$A'_t = \frac{A}{1+\exp(-\frac{A_t - B}{BC})} - \frac{A}{1+\exp(-\frac{B}{BC})}, \tag{4}$$

where A'_t is new media investment parameter included in the optimization model, A_t media investment, A - parameter which describes curve height, B - parameter which describes point, from where media investments become effective, and C shows curve growing speed and always is between 0.01 and 1.

Ad stock

Ad Stock or in some literature decay effect is important for Marketing Mix Models, because media investment is important not only for sales/visits but also for brand awareness. Some advertisements are memorable in subconscious and some people makes decisions some weeks later after advertisement they saw. For this reason there is Ad Stock effect.

Ad Stock effect is mathematically modelled and is expressed in terms of the "half-life" of the advertising. A "two-week half-life" means that it takes two weeks for the awareness of an advertising to decay to half its present level. Every advertisement has its own unique "half-life". Leone [5] have suggested half-life range around 7- 12 weeks, while industry practitioners typically report "half-lives" between 2-5 weeks, with the average for Fast Moving Consumer Goods (FMCG) Brands at 2.5 weeks.

Firstly Ad Stock effect was calculated from time series model with lags for TV Gross Rating Points (GRP) and when it was applied to the other media investment levels. Broadbent [6] suggested simple linear Ad Stock model:

$$A_t = T_t + \lambda A_{t-1}, t = 1, ..., n \tag{5}$$

where A_t is Ad Stock at time t, T_t is advertising variable at time t and λ is Ad Stock effect.

As equation (5) we could recursively substitute and expand to:

$$A_t = T_t + \lambda T_{t-1} + ... + \lambda^n T_{t-n}, t = 1, ..., n \tag{6}$$

and as λ is parameter from interval [0;1) so:

$$\lim_{n\to\infty} \lambda^n \to 0. \tag{7}$$

Fom equation (7) the conclusion that Ad Stock effect is decaying could be made.

Empirical model

Data

To apply in Section 1 described methods one of real media planning agency clients data was used. Due to confidentiality agreement with client there can not be mentioned client name. This data set is from related to actual shops visits which are measured by guest count. In model there are also included new shop openings, media spending from 10 different media channels and 17 differnt type campaigns, temperature, 8 competitors data and first/last day at school.

Shop visits (guest count) and new shop opening data was provided by client, competitors data is extracted from Kantar TNS database, first/last day at school was taken from Ministry of Education, Science and Sport and temperature data was taking from Vilnius airport database. Temperature was considered as significant variable which took place over seasonality, because there was made presumption that when weather is warmer more people go outside and visits shops. Also, the similar presumption was made about first and last day at school: because that days people are tend to celebrate and buy gifts.

For the model data were used weekly data from 5$^{\text{th}}$ January of 2015 till 11$^{\text{th}}$ February 2019.
R^2 was optimized for this equation:

$$\begin{aligned} GC_t = \beta_0 + \beta_1 Investments_t + \beta_2 new\_opnening_t + \beta_3 temperature_t \\ = \beta_4 FL\_school_t + \beta_5 Competitors_t + \epsilon_t \end{aligned}$$

There *Investments*:

$$Investments_t = Investments'_t + Ad\_Stock_1 Investments'_{t-1},$$

$$\begin{aligned} Investments'_t = \alpha_1 TV''_t + \alpha_2 Radio''_t + \alpha_3 Programmatic''_t + \alpha_4 Display''_t \\ + \alpha_5 VOD''_t + \alpha_6 Cinema''_t + \alpha_7 OOH''_t + \alpha_8 OOH\_Direct''_t \\ + \alpha_9 Magazines''_t + \alpha_{10} Other''_t. \end{aligned}$$

TV'', $Radio''$, $Programmatic''$, $Display''$, VOD'', $Cinema''$, OOH'', OOH_Direct'', $Magazines''$ and $Other''$ are S-shaped media investments summing all campaign in that media. For example:

$$TV''_t = \frac{A_1}{1 + \exp(-\frac{TV'_t - B_1}{B_1 C_1})} - \frac{A_1}{1 + \exp(-\frac{B_1}{B_1 C_1})},$$

$$TV'_t = \gamma_1 * Campaign_{1t} + ... + \gamma_{17} * Campaign_{17t}.$$

And *Competitors* are calculated from:

$$Competitors_t = Competitors'_t + Ad\_Stock_2 Competitors_{t-1},$$

$$Competitors'_t = \theta_1 Competitor_{1t} + ... + \theta_8 Competitor_{8t}.$$

So in this case there were 67 coefficients optimized.

Campaigns

In this period 17 different campaigns there were advertised. The biggest part of all these campaigns were advertised year after year. The same product type or event advertised is considered as the same campaign. So, the was import to understand which campaigns performs better results and has risen shop visits better than others.

Campaigns coefficient were choosen from interval [0;1]. Optimized campaigns coefficients results are in Table 1. From the table Table 1 the conclusions abount campaigns effectivity could be made. There is seen that $Campaign_2$ and $Campaign_8$ were the best performing campaigns, which has highest importance generating shop visits. In the mean time, $Campaign_9$, $Campaign_{14}$ and $Campaign_{17}$ were the worst performing campaigns and had just 35% effectivity comparing to the best campaigns.

Campaign	Coeficients
$Campaign_1$	0.66
$Campaign_2$	1.00
$Campaign_3$	0.56
$Campaign_4$	0.40
$Campaign_5$	0.65
$Campaign_6$	0.63
$Campaign_7$	0.44
$Campaign_8$	1.00
$Campaign_9$	0.22
$Campaign_{10}$	0.76
$Campaign_{11}$	0.63
$Campaign_{12}$	0.50
$Campaign_{13}$	0.61
$Campaign_{14}$	0.32
$Campaign_{15}$	0.83
$Campaign_{16}$	0.50
$Campaign_{17}$	0.35

Table 1: Campaigns coefficients

S-shapes

For each media it's own S-shape was calculated to get each media effectivity curve and understand effective level of investments. The parameter A uper limit is considered 125% of maximum weekly investment and lower limit 75% of maximum weekly investment. For the parameter B upper limit considered 125% of average weekly investment and lower limit 75% of average weekly investment. The parameter C values was chosen from real number interval [0.1;1]. The results for parameters A, B and C are in Table 2.

Media	A	B	C
TV	5496.27	1789.88	0.16
Radio	1542.17	156.72	1.00
Programmatic	4086.33	531.05	0.95
Display	1471.91	167.99	0.31
VOD	575.10	49.37	0.98
Cinema	766.43	44.09	0.94
OOH	4925.63	317.52	0.31
OOH_Direct	2097.57	897.35	1.00
Magazines	216.49	2.26	0.66
Other	1222.08	18.83	0.27

Table 2: S-shapes Coefficients

For TV the optimal weekly investment level 1800 - 2500 EUR. As parameter C is very small the curve in Figure 3 is growing fast and reaches its ssaturation point. Investments lower that 500 EUR is insufficient.

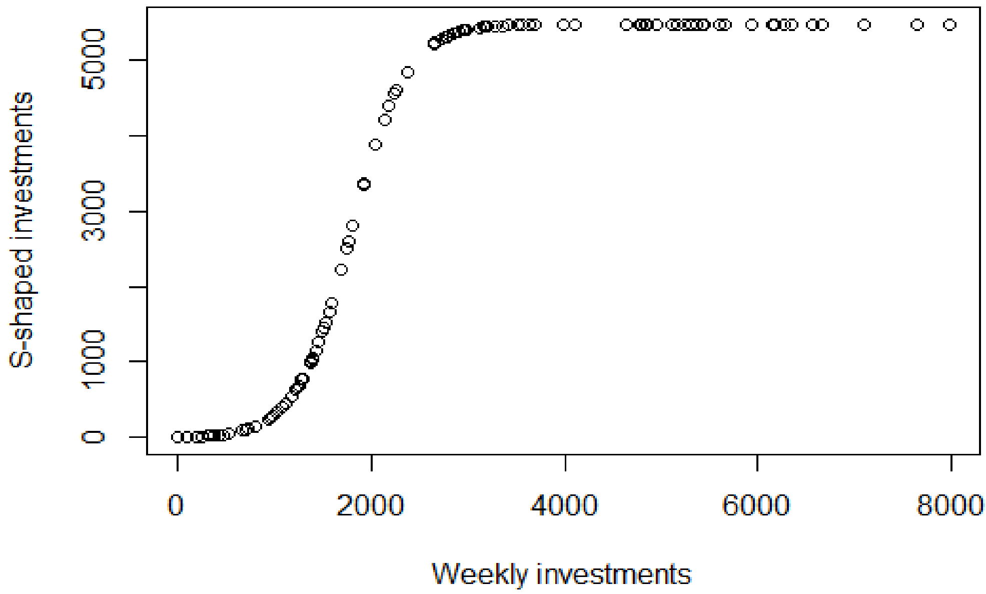


Figure 3: S-shape TV

For Radio the optimal weekly investment level is till 750 EUR. As parameter C is equal to 1 the curve in Figure 4 is growing slow and reaches its saturation point at 800 EUR.

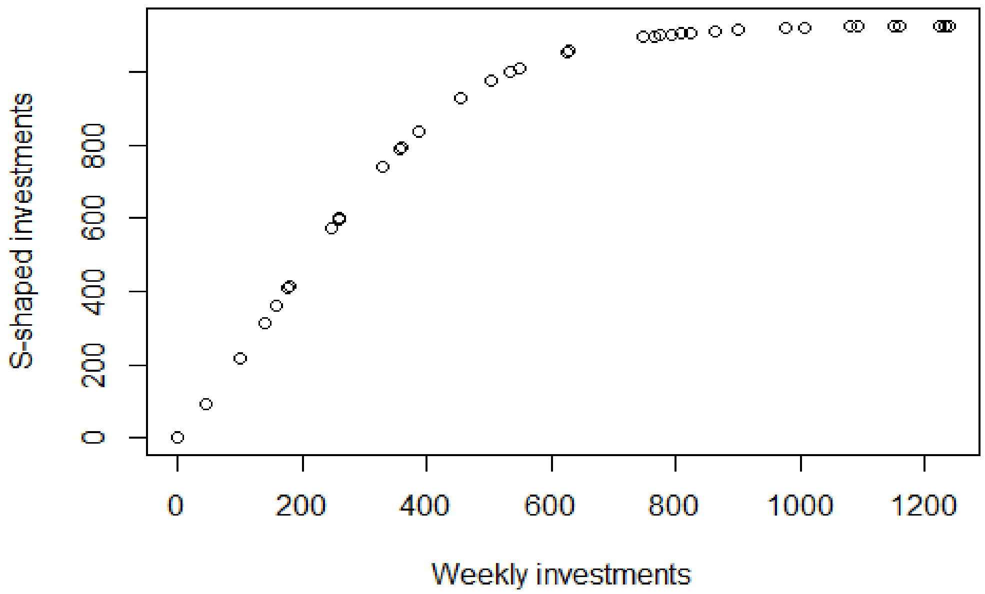


Figure 4: S-shape Radio

For Programmatic the optimal weekly investment level is till 2000 EUR. As parameter C is almost equal to 1 the curve in Figure 5 is growing slow and reaches its saturation point at 3000 EUR.

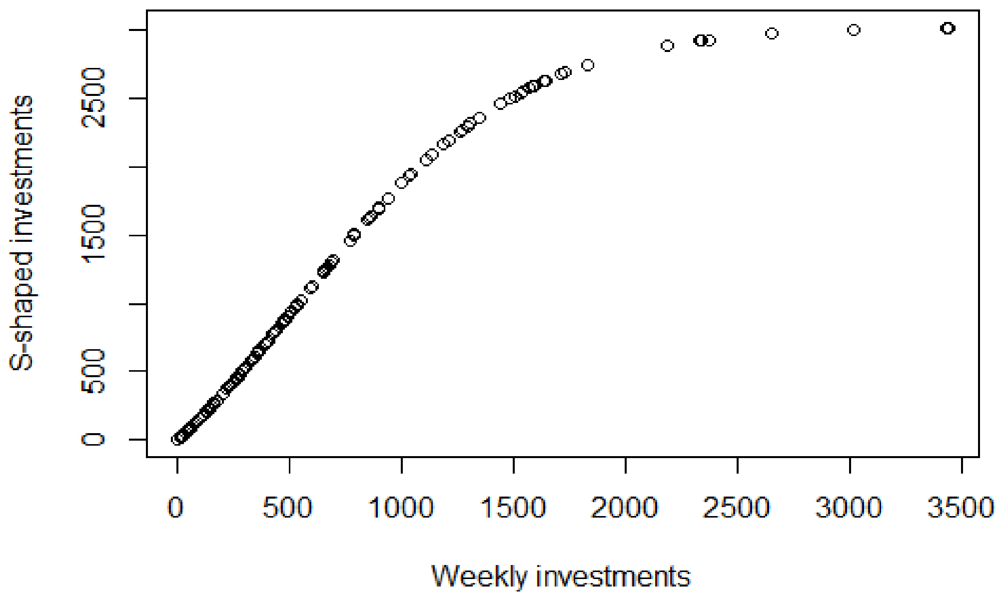


Figure 5: S-shape Programmatic

For Display the optimal weekly investment level is 100 - 400 EUR. As parameter C is small the curve in Figure 6 is growing fast and reaches its saturation point at 450 EUR. Investments under 100 EUR are insufficient for Display.

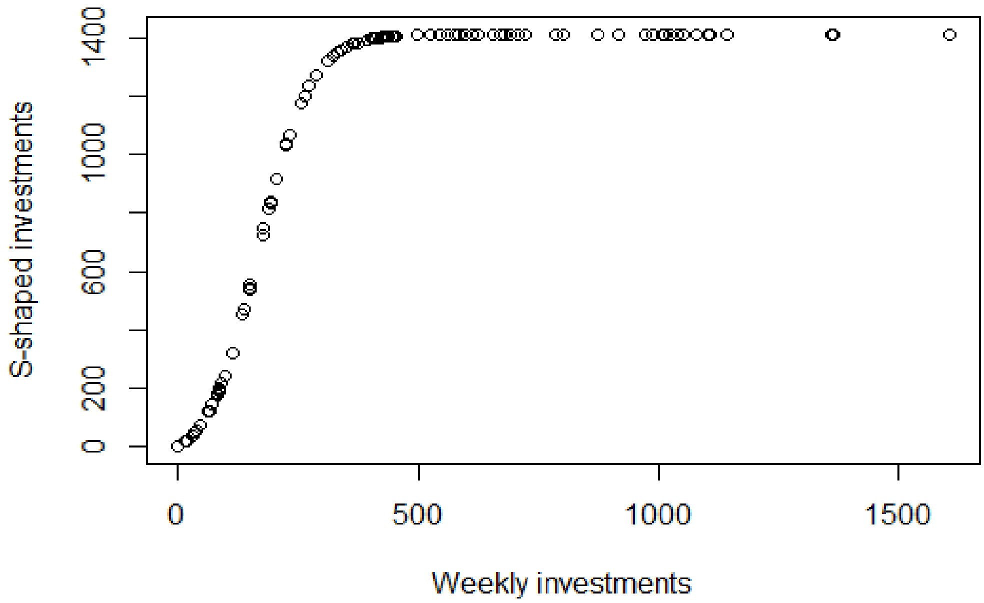


Figure 6: S-shape Display

For VOD the optimal weekly investment level is till 200 EUR. As parameter C is almost equal 1 the curve in Figure 7 is growing slow and reaches its saturation point at 200 EUR.

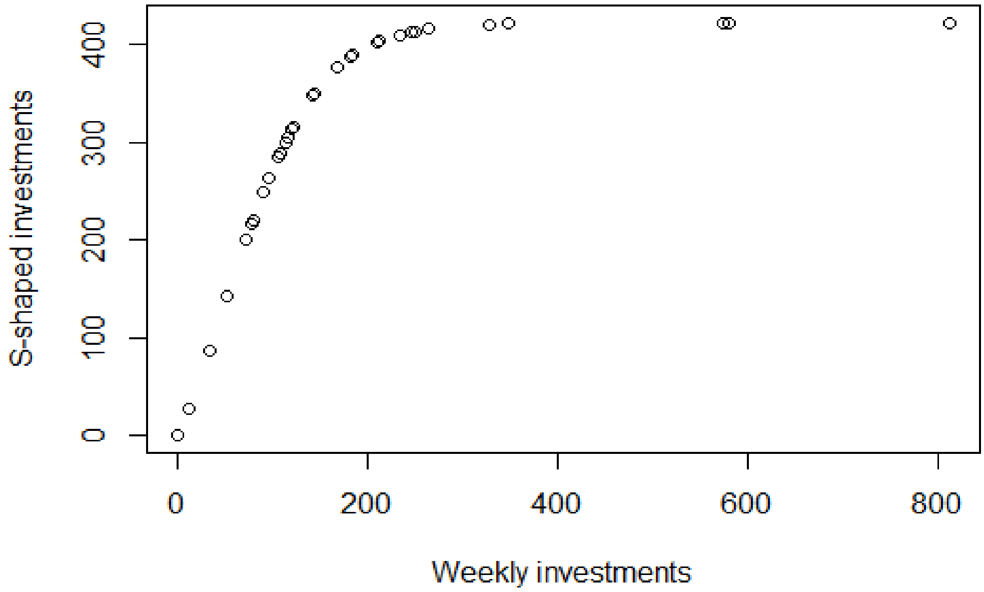


Figure 7: S-shape VOD

For Cinema the optimal weekly investment level is till 180 EUR. As parameter C is almost equal 1 the curve in Figure 8 is growing slow and reaches its saturation point at 180 EUR.

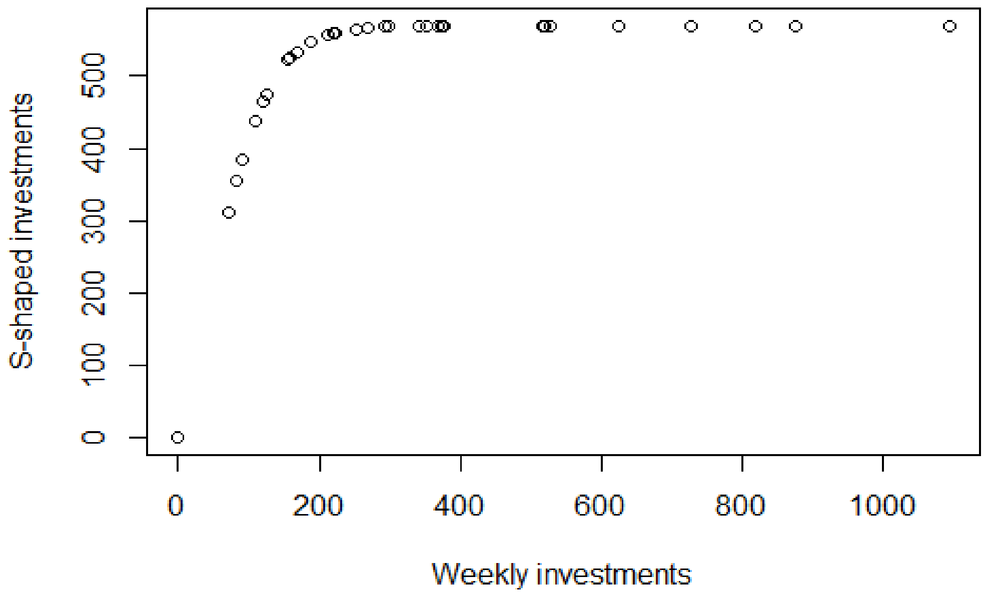


Figure 8: S-shape Cinema

OOH is the most overinvested media as the most of point are in right from saturation point. For OOH the optimal weekly investment level is till 1500 EUR. As parameter C is very small the curve in Figure 9 is growing fast and reaches its saturation point at 1800 EUR.

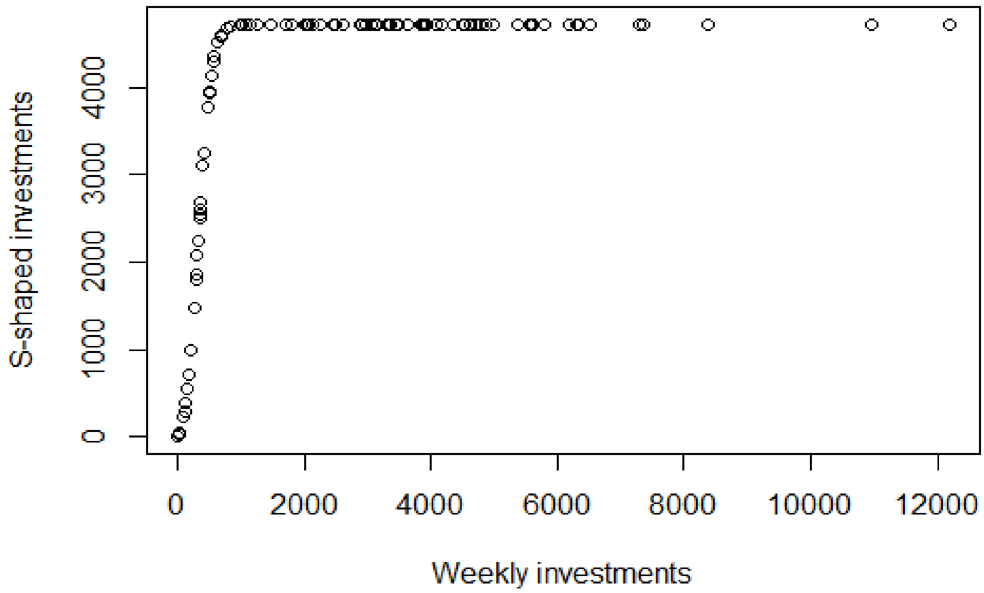


Figure 9: S-shape OOH

For OOH the optimal weekly investment level is till 2600 EUR. As parameter C is equal to 1 the curve in Figure 10 is growing slow and has no clear saturation point. However, for this media type has physical limitation because there is countable numbers of stand near to shops. But from this shape there is could be said that this media is very effectively planned.

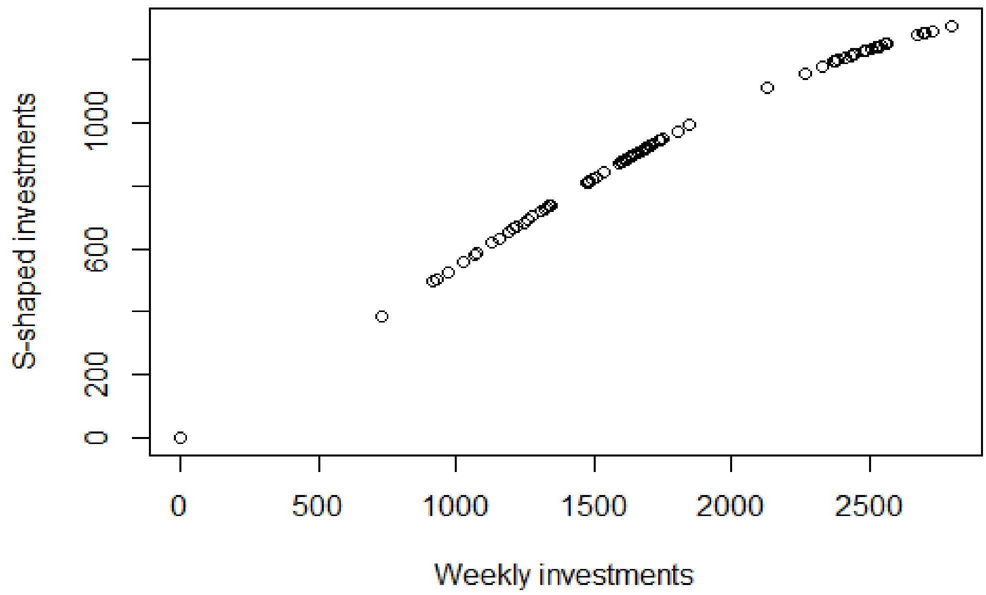


Figure 10: S-shape OOH Direct

For Magazines there could not be done any conclusions as only 3 times this media was historically used. This shown in the Figure 11.

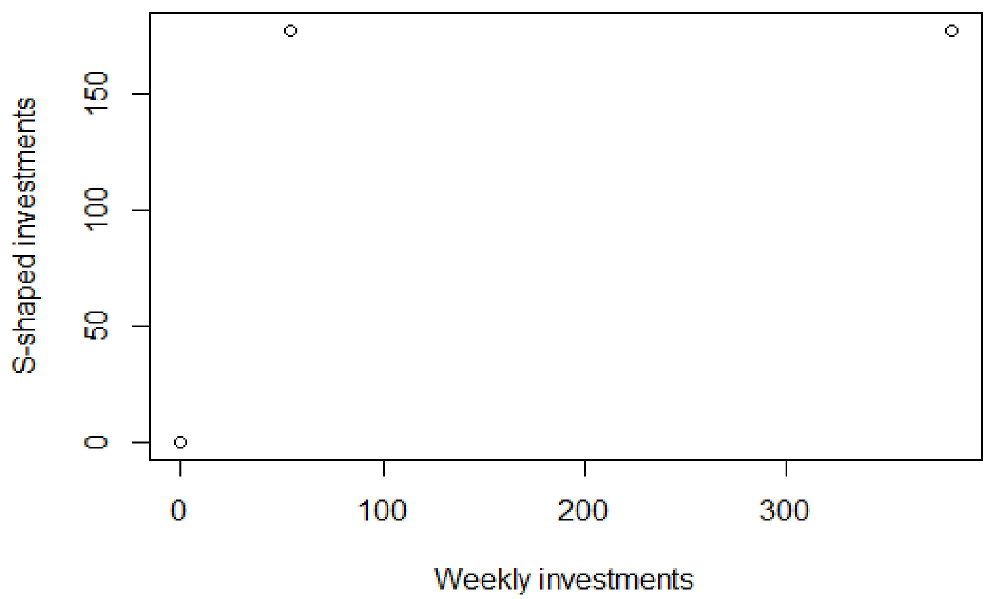


Figure 11: S-shape Magazine

For Other media types the optimal investment level is must be till 100

EUR. As parameter $C = 0.27$ the curve in Figure 12 is growing fast and reaches saturation point at 100 EUR.

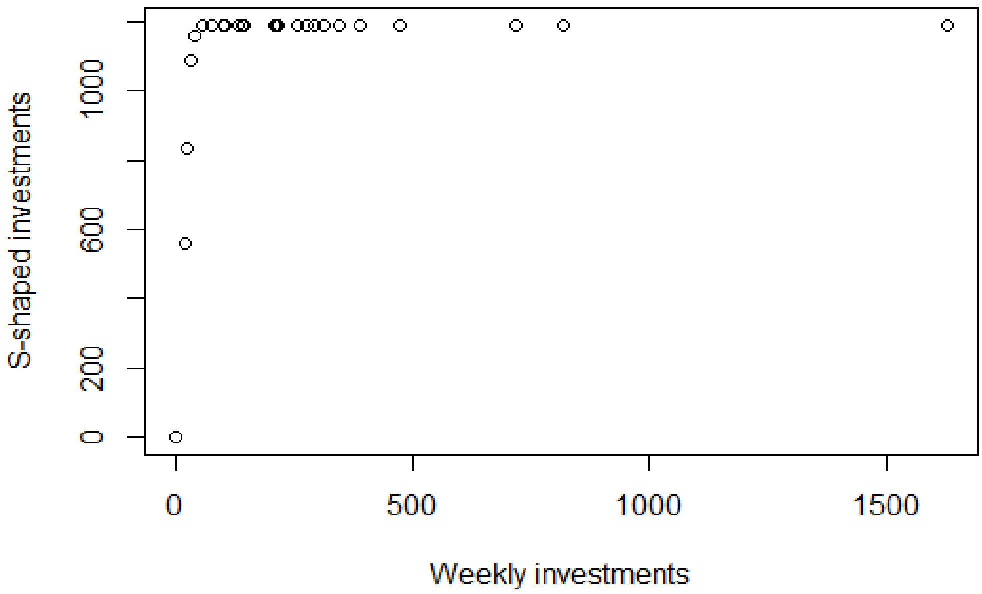


Figure 12: S-shape Other

Media

In this period there were 10 different media type used in marketing strategy. TV, OOH[2], OOH Direct[3] and Programmatic[4] had highest media spend from all media types. If we could make assumption that in next time periods there would be use the same campaigns in all media types, and there would be no overinvest from S-shapes then the most effective media types will would be Radio, Programmatic, Other and OOH Direct. The least effective media would be VOD, Display[5] and Cinema.

As there is different media type which has different investment levels for media coefficient were chosen from real numbers interval [0.5;2]. The optimized media coefficients values are in Table 3.

[2]OOH - outdoor stands

[3]OOH Direct - outdoor stands that directs to the shop

[4]Programmatic - digital media buying that utilizes data insights and algorithms to serve ads to the right user at the right time, and at the right price

[5]Display - digital media banners that is bouht in local portals and has current ads positions in the page

Media	Coefficient
TV	1.19
$Radio$	1.76
$Programmatic$	1.74
$Display$	0.76
VOD	1.01
$Cinema$	0.73
OOH	1.22
OOH_Direct	1.32
$Magazines$	1.25
$Other$	1.61

Table 3: Media coefficients

Competitors

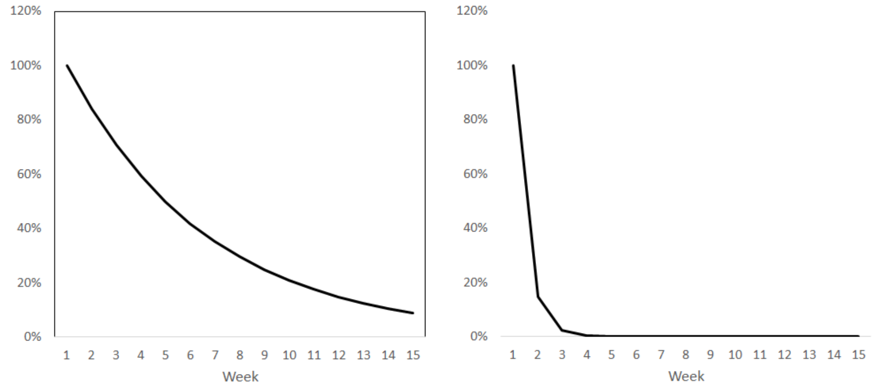
In the model were included 9 competitors. The was several competitors which are direct has the same goods in the store as client and some of them has only the majority of main goods as in the client. The coefficients for the clients were chosen from Interval [-1;1], because some competitors could have positive effect to the client as category drivers and some negative effect. From competitors only one of them had the same investment level as client other had 3 or 5 times lower investment level. From coefficient optimization competitors coefficients are in Table 4.

Competitor	Coefficient
$Competitor_1$	-0.82
$Competitor_2$	0.00
$Competitor_3$	0.01
$Competitor_4$	0.59
$Competitor_5$	0.73
$Competitor_6$	0.21
$Competitor_7$	0.87
$Competitor_8$	0.77

Table 4: Competitors coefficients

Ad Stock effect

The media investment and competitors Ad stock coefficients were chosen from real numbers intervals [0;1]. After optimization procedure to media investment Ad Stock coefficient was chosen 0.84 which means that for next week media investment has 84% of invested value from current week. This percent is considered normal for old brands. For competitors investments Ad Stock is much shorter - just 15%. The Ad Stock Effect on weekly basis is shown in Figure 13.



(a) Media Ad Stock (b) Competotors Ad Stock

Figure 13: Ad Stock effect

Model explanation

The optimization process optimized model R^2 to 0.87 which means that, whith optimized all 67 parameters, model describes visits to shop very well. The coefficients for model equation presented in the table Table 5.

$$GC_t = \beta_0 + \beta_1 Investments_t + \beta_2 new\_opnening_t + \beta_3 temperature_t \\ + \beta_4 FL\_school_t + \beta_5 Competitors_t + \epsilon_t$$

Coefficient	Value	Std. error	p-value
β_0	26710	3725	0.00
β_1	0.14	0.02	0.00
β_2	4403	392.7	0.00
β_3	555.7	39.43	0.00
β_4	6401	1578	0.00
β_5	0.16	0.03	0.00

Table 5: Model Coefficients

All variables are significant as $p - value$ for all coefficients are less than 0.05, so the $H_0 : \beta_0, ..., \beta_5 = 0$ could be rejected.

Even coefficient for media investments is quite small comparing to other, but media investment influenced 8% of shop visits during all the time. This number is quite high as the client is old brand in Lithuania so from Sharp [8] theory this number is normal. New shops opening generated additional 12% of shops visits, competitors advertising influence 2% of shop visits, first and last day at school during all period influenced 0.3% visits and 5% of visits were generated due to temperature or in other words seasonality.

Another important question from the model results: which media generated the most visits. From Figure 14 is seen that OOH, TV, Programmatic and OOH Direct generated the biggest part shop visits from all media types. In Figure 14 "Base" is all other variables and Intercept.

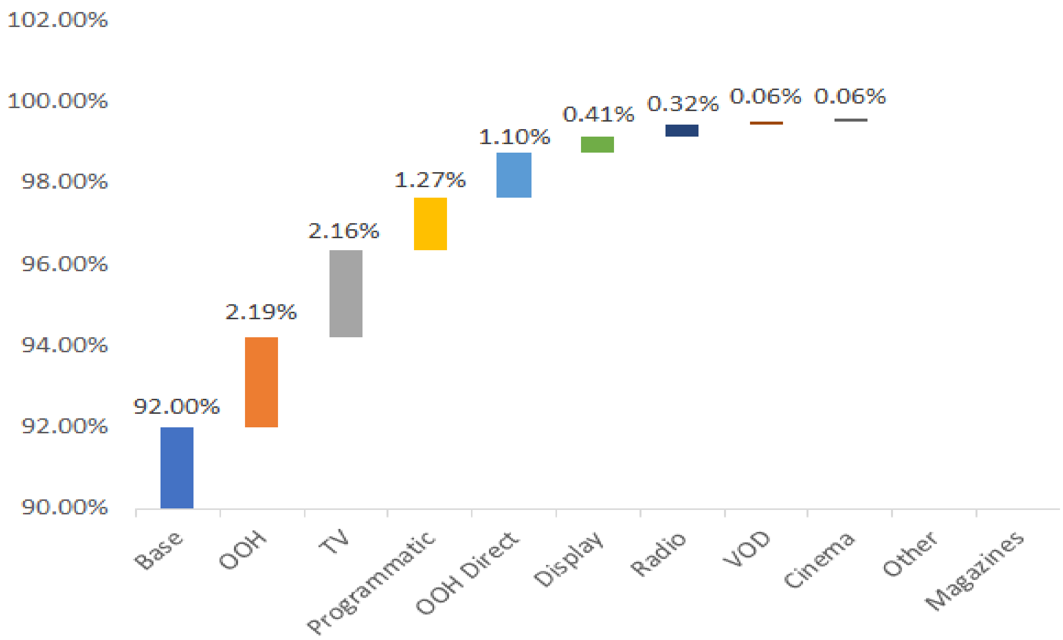


Figure 14: Model explanation

Comparing media effectivity by price per visitor from each media Figure 15 is seen that Other, OOH, Programmatic, Radio, VOD and TV are the cheapest media, price per visitor from each media is less than 1.5 EUR. And the most expensive media are magazines.

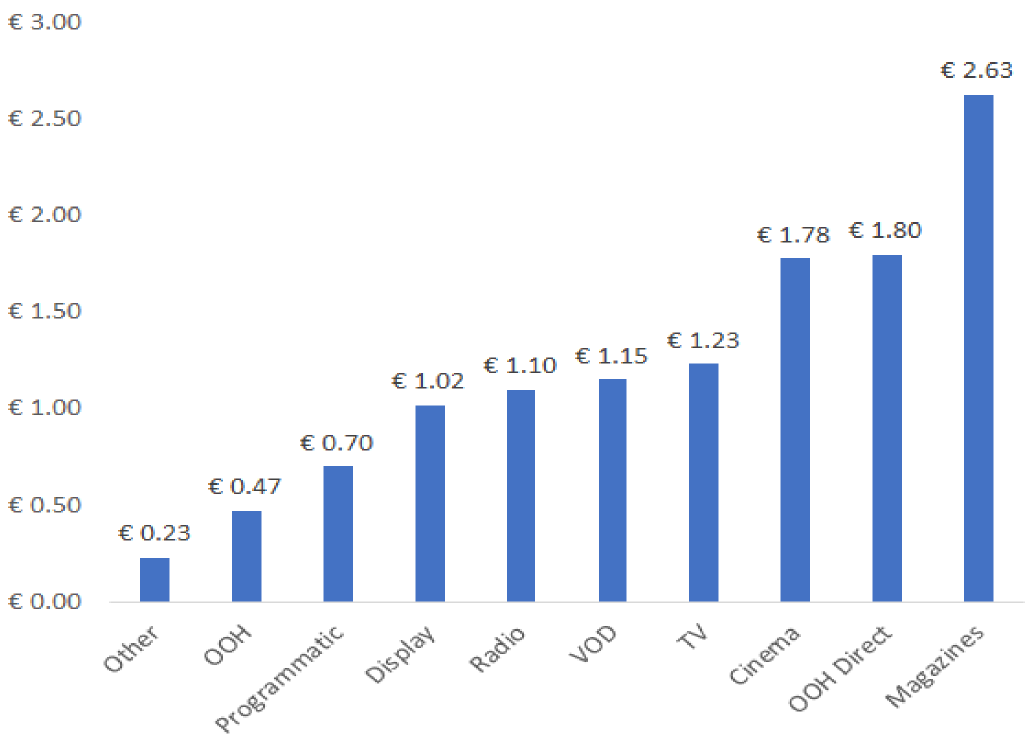


Figure 15: Media price

Comparing results from media coefficients, S-shapes, Figure 14 and Figure 15 the conclusion of media split could be done. So the must have media in all campaigns should be Programmatic, TV, OOH and Radio. All others media are less important should be used as additional media type for strengthening campaigns. Also, magazines and cinema should be considered as rejected media types.

Model error checking

The last step of checking modelling goodness is to check if model errors are normal. This was done by QQ plot Figure 16. From Figure 16 there could not be made conclusion that error are not normal because both sets of quantiles came from the same distribution forming the straight line. Also Durbin - Watson test was performed where:

$$H_0 : \rho = 1, H_1 : \rho = 0. \tag{8}$$

From Durbin - Watson test $autocorrelation\ coefficient = 0.3$, $Durbin - Watson\ statistics = 1.26$ and $p - value = 0.0$. So the H_0 was rejected and model residuals are uncorrelated and model could be used.

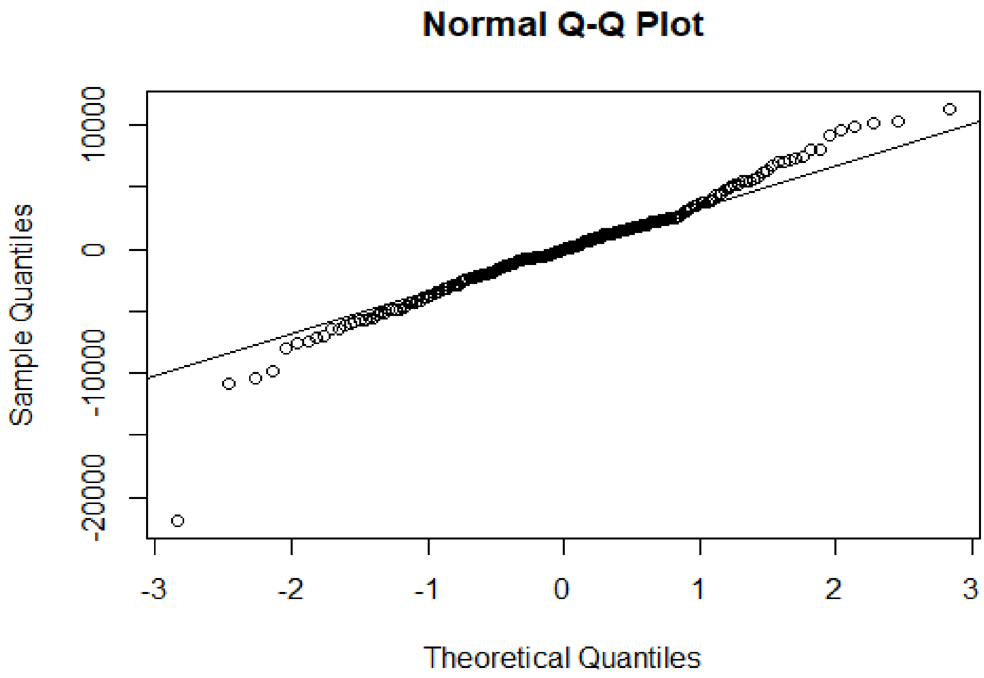


Figure 16: QQ plot

Media optimization suggestions

When we proved that our model is well described data and is consistent. The further step is demonstrating how to split different weekly budget to media. Assuming that all campaigns are the same good (this is done because media planning agency could not change campaigns this decision is made only from the client side). Media split were optimized with parameters from model optimizing generated shop visits.

There was 2 different weekly media budget optimized. In Table 6 is chosen average weekly budget and in Table 7 high weekly media budget is chosen:

Media	Budget
TV	2000 EUR
Programmatic	1900 EUR
OOH	900 EUR
Radio	900 EUR
Display	500 EUR
VOD	450 EUR
Cinema	350 EUR
Total	7000 EUR

Table 6: Average week budget

Media	Budget
Programmatic	4500 EUR
TV	4250 EUR
Radio	1500 EUR
OOH	1300 EUR
Display	650 EUR
VOD	450 EUR
Cinema	350 EUR
Total	7000 EUR

Table 7: High week budget

Conclusions

Where was combined Media Mixed Marketing Models methodology applied using optimization of R-square. Multiple parameters were solved during the optimization process. There was proved that the model is consistent. Optimization procedure which ran several times always get the same parameter values. This methodology could be used solving similar problems.

For particular client case we get that the best performing media was Programmatic, TV, OOH and Radio. The best performing campaigns were $Campaign_2$ and $Campaign_8$. In this case competitors had no very small positive impact to the client. In total media explained 8% of all visits. But as the client is old brand in Lithuania, we could see that it has long term ROI 26710 weekly visits.

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