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On a classification of word problems from the first grade Lithuanian textbooks

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Abstract. Word problems are classified to S problems and P problems by Verschaffel [10], classification is being specified and expanded. Reviewed word problems in Lithuanian first grade textbooks and divided to types. Submitted recommendations to use more varied types word problems, that would let to expand concepts understanding, develop mathematical reasoning, motivate to study word problem.

Keywords: word problems; textbooks; P problem; S problem; mathematical reasoning

AMS Subject Classification: 03D40, 97E50, 97-11

Introduction

If student has to choose which way to take going from school he chooses it and is able to justify his decision. If he has the same word problem in textbook he stops thinking, stops talking and just gives his teacher a number. Why do we loose the main point in mathematics – which is reasoning? We took a closer look to Lithuanian textbooks to see what word problems we can find in them. To do that we reviewed and customized word problems classification for Lithuanian first grade textbooks. In the end of the article we will make suggestions how word problems in the textbooks could be improved.

Analyzed word problems

We analyzed James Stigler and others article "An Analysis of Addition and Subtraction Word Problems in American and Soviet Elementary Mathematics Textbooks"

[7] and decided to make similar analysis for addition and subtraction word problems in Lithuanian textbooks. They used such classification of word problems as shown in Table 1. In order the word problem to be coded it had to present information and a question. Authors studied word problems containing only text. We also included problems with some images instead of text.

The structure of a word problem (WP) is formed of the following three parts:

1. Description of the context of the WP;
2. Information needed to answer the question;
3. The question.

The second and the third parts are necessary for the word problem to be correctly formulated. We say that the information supplied for the word problem is optimal if it is necessary and sufficient to answer the question.

Problems in word problems

Some problems are standard where students can take numbers and do some operations with them, and some are different. So we divided word problems to S problems and P problems. We found this classification used in 1994 by Vershaffel *et al.* [9]. They describe these problems as standard and problematic. We specified definitions and sorted them into several types (see Fig. 1).

Definition 1. We say that the WP is *S problem* if it has optimal information needed to answer the question. If the S-problem can be solved using just one step then it is called *S1 problem*, otherwise it is called *S2 problem*.

We divided S1 problems into 20 types from [7] (Table 1). It is just one example of many classifications, which can show what word problems we have.

We call more problematic word problems P problems.

Definition 2. We say that the WP is *P problem* if it has either more than needed sufficient information or has less than needed necessary information to answer the question. In the first case we call the WP as *P1 problem* in the second case we call the WP as *P2 problem*.

It could be problems where students have to choose which information is important, which is not or given information is not clear and need to be discussed. So students sometimes have to choose solution models or analyze different cases. We divided P problems to P1 and P2 problems. These problems require mathematical reasoning and finding your own strategies.

Examples of P problems

Lets see how P problems look like and why we need them.

Saljo and Wyndhamn [5] did study based on two exercises (S problem and P problem) A) and B).

A) A cow produces 18 liters of milk per day. How many liters of milk does the cow produce during one week?

Table 1. S1 problems types by action.

Join	**Separate**
Change	
1. Goda had 5 stickers. Adas gave her 8 more stickers. How many stickers does Goda have altogether?	2. Goda had 13 stickers. She gave 5 stickers to Adas. How many stickers does she have left?
3. Goda has 5 stickers. How many more stickers does she need to have 13 stickers altogether?	4. Goda had 13 stickers. She gave some to Adas. Now she has 8 stickers left. How many stickers did Goda give to Adas?
5. Goda had some stickers. Adas gave her 5 more stickers. Now she has 13 stickers. How many stickers did Goda have to start with?	6. Goda had some stickers. She gave 5 to Adas. Now she has 8 stickers left. How many stickers did Goda have to start with?
Combine	
7. Goda has 5 red stickers and 8 blue stickers. How many stickers does she have?	8. Goda has 13 stickers. Five are red and the rest are blue. How many blue stickers does Goda have?
Compare	
9. Goda has 13 stickers. Adas has 5 stickers. How many more stickers does Goda have than Adas?	10. Goda has 13 stickers. Adas has 5 stickers. How many fewer stickers does Adas have than Goda?
11. Adas has 5 stickers. Goda has 8 more than Adas. How many stickers does Goda have?	12. Adas has 5 stickers. He has 8 fewer stickers than Goda. How many stickers does Goda have?
13. Goda has 13 stickers. She has 5 more stickers than Adas. How many stickers does Adas have?	14. Goda has 13 stickers. Adas has 5 fewer stickers than Goda. How many stickers does Adas have?
Equalize	
15. Goda has 13 stickers. Adas has 5 stickers. How many stickers does Adas have to win to have as many stickers as Goda?	16. Goda has 13 stickers. Adas has 5 stickers. How many stickers does Goda have to lose to have as many stickers as Adas?
17. Adas has 5 stickers. If he wins 8 stickers, he will have the same number of stickers as Goda. How many stickers does Goda have?	18. Adas has 5 stickers. If Goda loses 8 stickers, she will have the same number stickers as Adas. How many stickers does Goda have?
19. Goda has 13 stickers. If Adas wins 5 stickers, he will have the same number of stickers as Goda. How many stickers does Adas have?	20. Goda has 13 stickers. If she loses 5 stickers, she will have the same number stickers as Adas. How many stickers does Adas have?

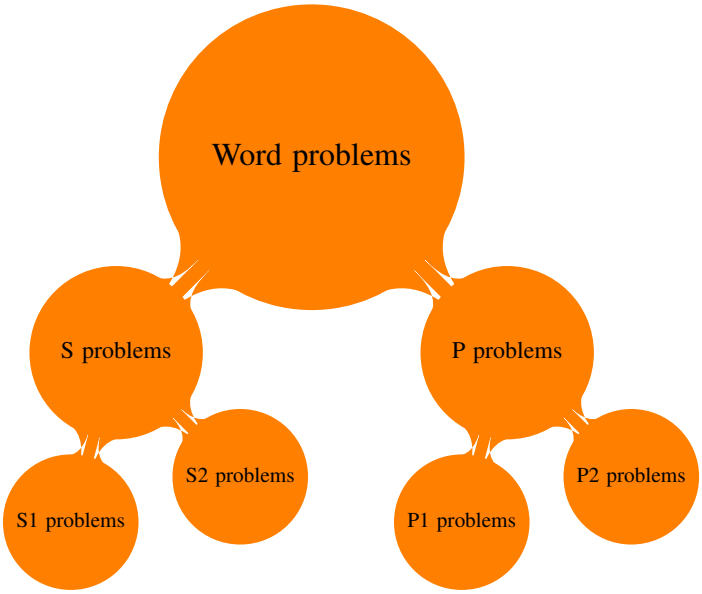

Fig. 1. Word problems classification.

B) Kalle goes to school and on average he has seven lessons per day. How many lessons does he have per week?

Problem A) is S problem. To solve it students just have to multiply 18 by 7 (which is a number of week days). B) problem is P1 problem, to solve it students have to multiply 7 (lessons) by 5, not by seven. In the text there is no information, that students go to school from Monday to Friday. All students know that, so they are able to use this information. But results show that its a harder problem. There were 90% correct answers to A) problem ant 70% correct answers to B) problem.

There is an interesting P2 problem example which is as follows:

> Two boys, Charles and Martin, are going to help Nicholas rake leaves on his plot of land. The plot is 1200 square meters. Charles rakes 700 square meters during four hours and Martin does 500 square meters during two hours. They get 180 crowns for they work. How are the boys going to divide the money so that is fair?

There are results in Table 2. We can see that one correct answer does not exist, but kids discussed different models of solution.

We can see that students discuss, try different models, search for solutions when they solve P2 problems.

Lithuanian textbooks

We studied three first grade textbooks: “Skaičių šalis”, B. Balčytis, A. Vaičiulienė, 1992. “Matematika 1 klasei”; A. Kiseliovas, D. Kiseliova, 2016. “TAIP!”; Rita

Table 2. Models suggested for sharing money in first and final suggestion from the groups.

Model for sharing	First suggestion No. of groups	Final suggestion No. of groups
I. Divide equally (180/2)	11	0
II. Amount of work	9	3
III. Time worked	6	14
IV. Piece rate/payment by performance	0	7

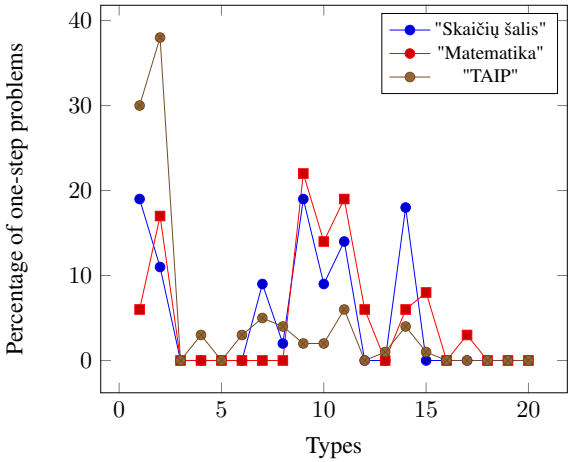

Fig. 2. Distribution of one step S problems according to type.

Rimšienė, Ada Kavaliauskienė, Linas Vilčinskas, Šviesa, 2018. Me and Rimas Norvaiša studied these three textbooks separately and then compared results and came to an agreement for each task type. You can see classified S problems in Fig. 2. As you can see there are no word problems for some types.

We found some P problems in these textbooks. In textbook "TAIP!" there were two P1 problems of 167 word problems, others were S problems. In textbook "Matematika I" were 6 P1 problems and 1 P2 problem of 44 problems. So there were 16% P problems. All of them were marked as harder, or more interesting problems. In textbook "Skaičių šalis" there were no P problems.

One of P1 problems we found in [4]:

> Kajus had 16 nuts. Gerda had 15 nuts. Kajus gave 1 nut to Gerda. Do they have the same amount of nuts?

It could seem, that it is simple S1 problem. But then student could just add one to 15 (Kajus gave 1 nut to Gerda) and equalize it to 16 (Kajus had 16 nuts) and say they have the same amount of nuts. In this problem there is some information hidden. When Kajus gave one nut to Gerda, he ends up with one nut less. Students should not just add 1 to 16, they also have to subtract 1 from 16. Then they will get an answer that kids do not have the same amount of nuts.

P2 problem found in [3]:

100 chestnuts were pour out from two boxes. How many chestnuts could have been in each box?

It is not enough information in problem to get one answer, so student have to give set of answers.

What is needed to be changed

More S problems' types

Talking about S problems Vergnaud [8] explains how concept's meaning comes to students: "concepts meaning does not come from one situation only but from a variety of situations". For example addition can be met in situations of combining, changing, comparing and equalizing. So if we let students solve just some of these situations, their understanding of concept will be just partial. As we can see there are a lot of first and second types S problems in textbooks. Quite a lot of 9, 10, 11, 14 types and some types we do not have at all. So textbook authors should pay more attention distributing word problems types.

Let students solve P problems

Due to Verschaffel *et al.* [10] the word problems in textbooks often are like that:

- every problem is possible to solve;
- all information given should be used;
- there is only one possible solution and only one correct answer.

Authors stress that "socialisation into such tradition for interpreting problems makes it difficult to realize what particular problem is about". We can see that there are a lot of S problems and just few P problems. Students get used to look to numbers and decide what schema to use. They do not think what a text story gives us, what a problem is about and when they get different problems, (new type, or in new topic) they can not solve it if they do not know schema. Due to Hatano [2] we should try to make distinction between "understanding by schema application" and "understanding through comprehension activity". We suggest that in textbooks should be not just S problems, but also more P problems to teach students read text story, understand it and make decisions.

Wyndhamn and Saljo in [11] show how problematic problems develop mathematical reasoning. They say that S problems "seem to indicate lack of capacity or logical reasoning ability". And P problems by themselves have reasoning requests. If we say that mathematics develop reasoning and we want kids to understand, estimate, think, not just give an answer then we should have more P problems in our textbooks.

Conclusions

To connect real life with mathematics and also develop mathematical thinking in schools we should use not just standard problems, but also problematic problems. Then students act and become involved in problems. When they have only S problems and the same type of problems again and again they stop thinking and just do

arithmetics. They also do not get full concept meaning (of addition, subtraction). In Lithuanian textbooks we should use more different problems (P problems, other S problems types) and invite students to think and make reasoning.

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REZIUMĖ

Apie žodinių uždavinių klasifikaciją pirmos klasės vadovėliuose

I. Kilienė

Žodiniai uždaviniai klasifikuojami į S problemas ir P problemas pagal Verschaffel [10], klasifikacija patikslinama ir išplečiama. Apžvelgiami žodiniai uždaviniai pirmos klasės vadovėliuose Lietuvoje ir suskirstomi į tipus. Pateikiamos rekomendacijos naudoti įvairesnių tipų žodinius uždavinius, tai leistų formuotis platesniam sąvokų suvokimui, ugdytų matematinį samprotavimą, skatintų nagrinėti uždavinio problemą.

Raktiniai žodžiai: žodiniai uždaviniai; P problemos; S problemos; vadovėliai; matematinis samprotavimas